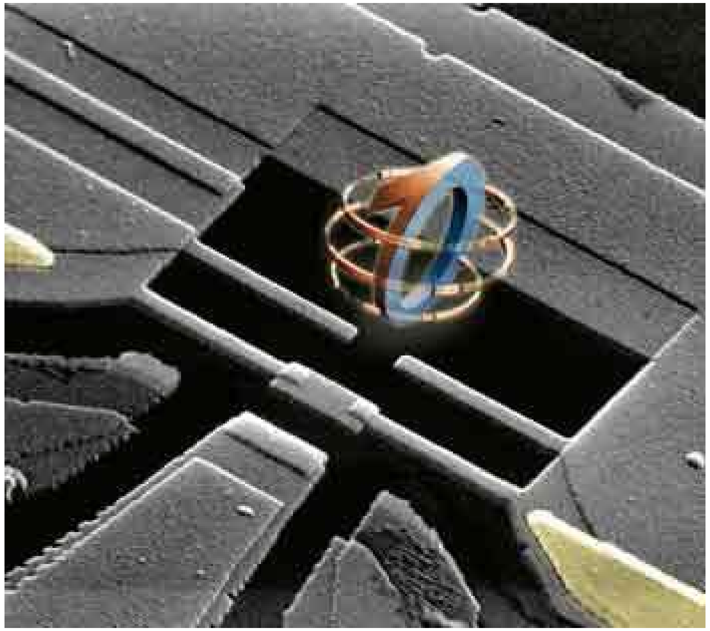
New Advances in QIS and Technologies , Samarkand, 10-18 September, 2019

Josephson junctions based on novel compounds for superconducting qubits



Iman N. Askerzade

Computer Enginering Dept. of Ankara University, Turkey

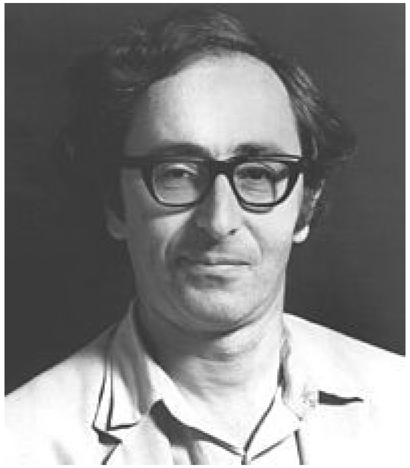
and

Institute of Physics Azerbaijan National Academy of Sciences, Azerbaijan

Layout of presentation

- Josephson effects : basic equations
- New superconducving materials and the influence on current-phase ralation of JJ
- Superconducting qubits
- Spectrum of Josephson qubits with anharmonic CPR
- Conclusions

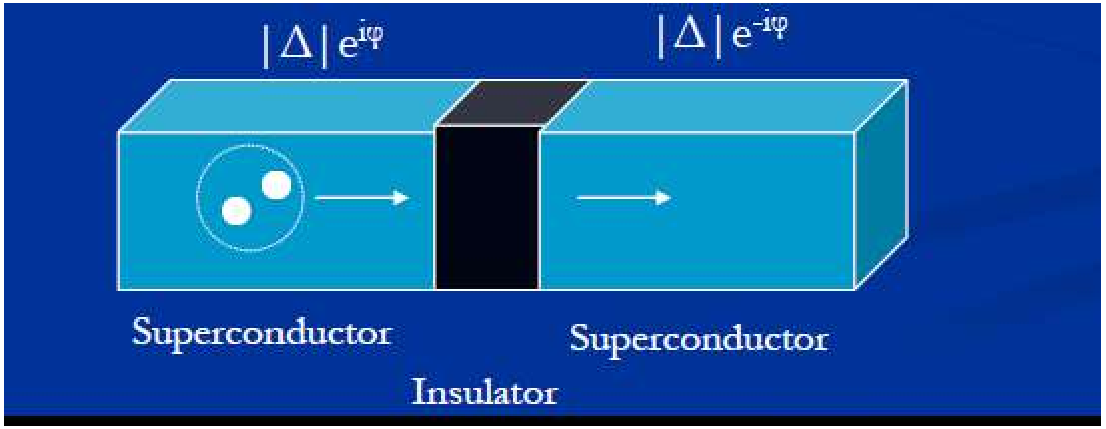
This study suported by TUBITAK Project 118F093.



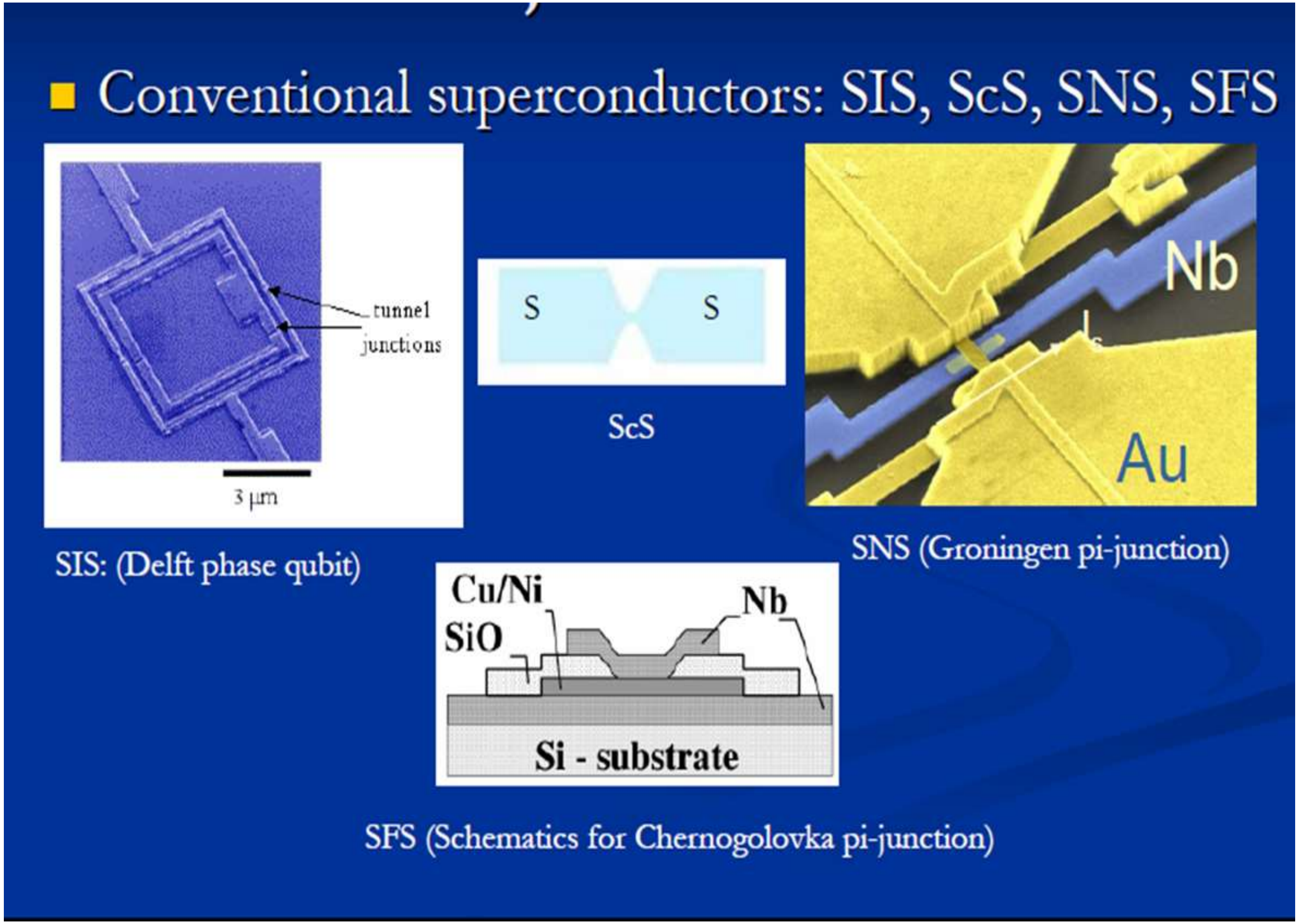
1. Josephson effect

B. Josephson (1962)

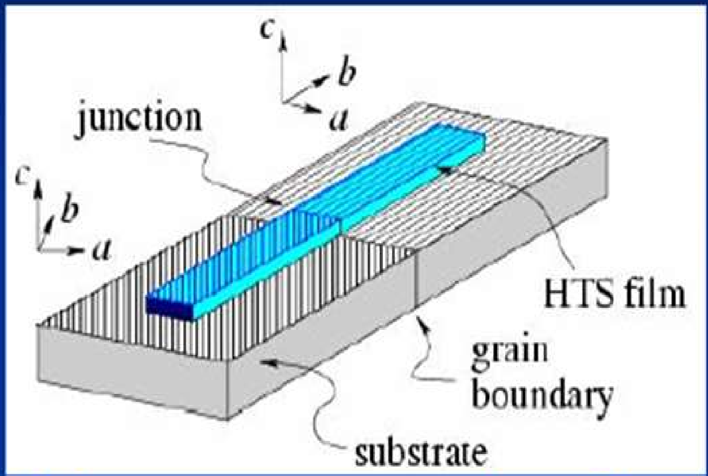
- Two superconductor or two condensates with macroscopic wave functions



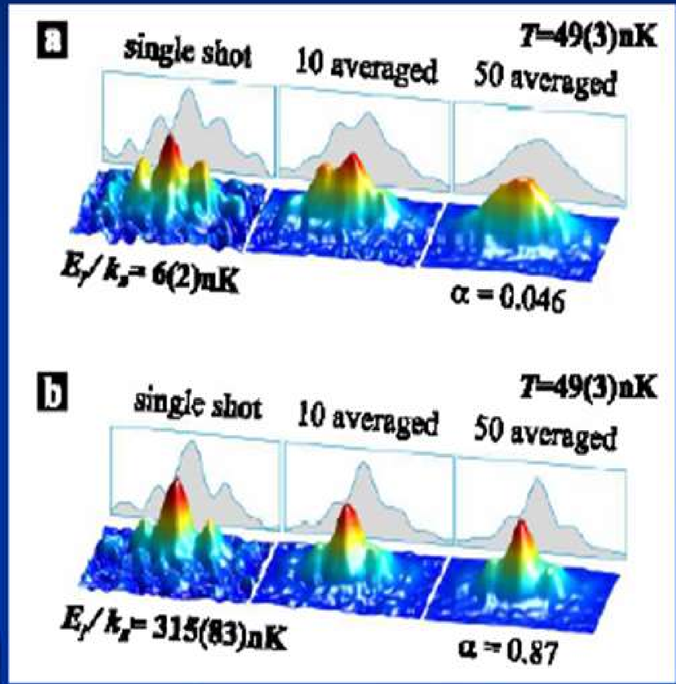
Conventional Josephson junction types



Josephson effect in different systems

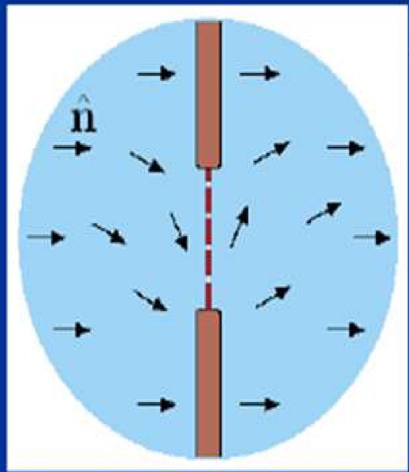


High-temperature superconductor JJ's



BEC: Gati, et al., cond-mat/0601392

Helium superfluid JJ: pinhole arrays (from E. Thuneberg)



Gati Phys. Rev. Lett. 95, 010402,2005

Method of tunnel Hamiltonian

$$H = H_1 + H_2 + \hat{T},$$

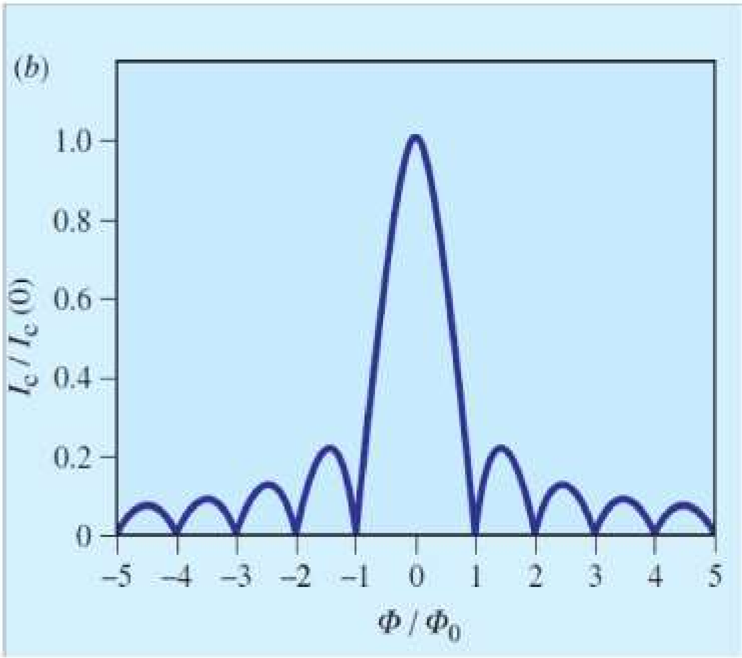
$$\hat{T} = \sum_{\mathbf{pq}\sigma} (T_{\mathbf{pq}} a^{+}_{\mathbf{p}\sigma} a_{\mathbf{q}\sigma} + T^{*}_{\mathbf{pq}} a^{+}_{\mathbf{q}\sigma} a_{\mathbf{p}\sigma})$$

$$I = I_c \sin\phi; \;\; \phi = \phi_1 - \phi_2,$$

$$I_c = \frac{\pi\Delta}{2eR_N} \tanh\frac{\Delta}{2T},$$

Influence of external magnetic field on the critical current

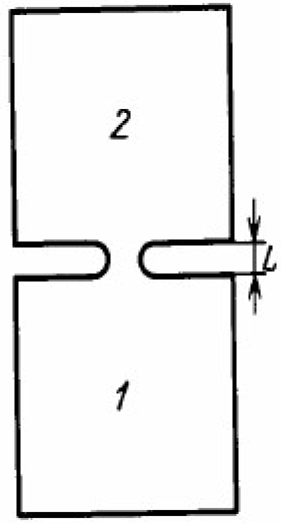
$$I_c(H) = I_c(0)\left|\frac{\sin(\frac{\pi\Phi}{\Phi_0})}{\frac{\pi\Phi}{\Phi_0}}\right|$$



ScS junctions: Aslamazov-Larkin (1970)

Ginzburg-Landau (GL) equation

$$-\xi^2\nabla^2\psi - \psi + \psi^3 = 0$$



$$|\Psi_0|^2 = -\frac{2\alpha}{\beta} = \frac{2\gamma(T_c - T)}{\beta}$$

$$\xi^2(T) = \frac{\hbar^2}{4m\gamma(T_c - T)}$$

$$I = \frac{\alpha\hbar e}{\beta m}\,\mathrm{Im}(\psi^*\psi) = I_c \sin\phi;\ I_c = \frac{\alpha\hbar e}{\beta m}$$

Bogolyubov-De-Gennes equations

$$\left\{\frac{\hat{p}^2}{2m} - \mu\right\} u(r) - \Delta(r) v(r) = \varepsilon u(r)$$

$$\left\{\frac{\hat{p}^2}{2m} - \mu\right\} v(r) + \Delta^*(r) u(r) = -\varepsilon v(r)$$

$$E_J = \Delta\sqrt{1 - D\sin^2(\phi/2)}$$

$$I(\phi) = \frac{2\pi}{\Phi_0}\frac{\partial E_J}{\partial \phi}$$

$$I(\phi) = \frac{e\Delta^2}{2\hbar}\sin\omega\sum_{n=1}^{N}\frac{D_n}{E_n}\tanh\frac{E_n}{2T}$$

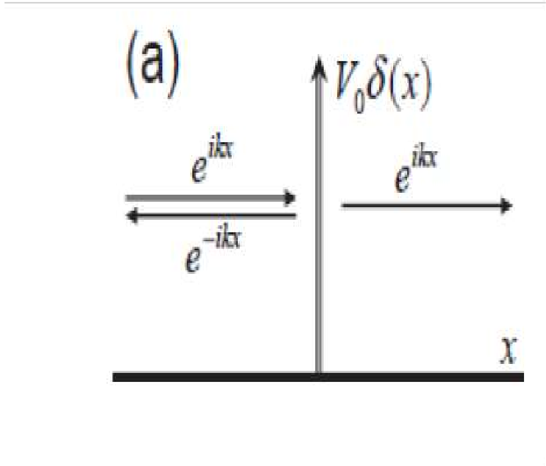
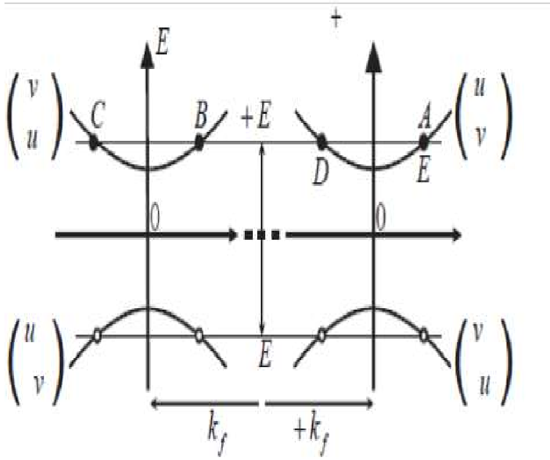


Figure 4:



Basic equation of JJ with conventional CPR

1. Basic properties

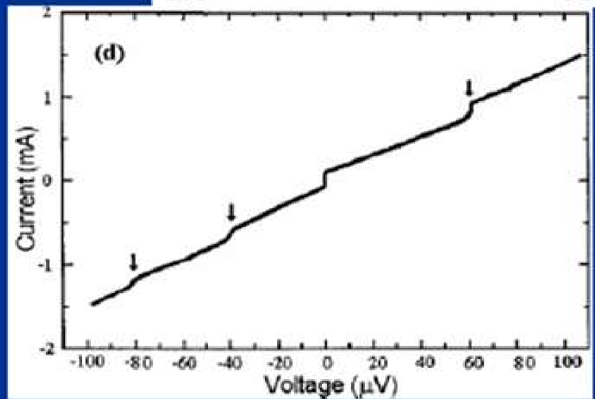
- Ac/dc Josephson relations

$$I = I_C \sin(\varphi_1 - \varphi_2)$$

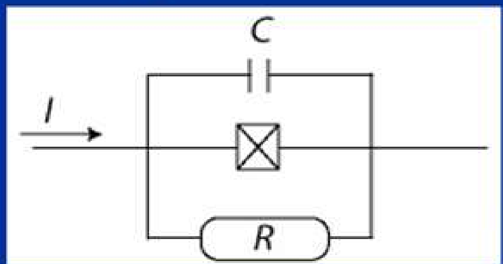
$$\frac{d}{dt}(\varphi_2 - \varphi_1) = \frac{2eV}{\hbar}$$

- Shapiro steps

$$V = V_0 + V_1 \cos(\omega_1 t) \longrightarrow$$



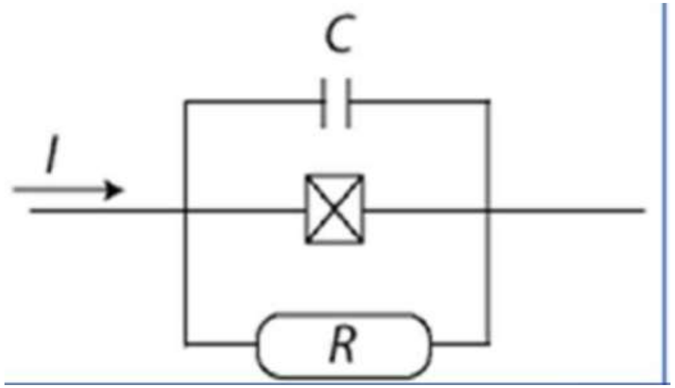
- RCSJ model



$$I = I_c \sin(\varphi) + \frac{\hbar}{2e}\dot{\varphi}/R + C\frac{\hbar}{2e}\ddot{\varphi}$$

Basic equation of JJ with conventional CPR

$$C\frac{dV}{dt}+\frac{V}{R}+I_c\sin\phi+I_F=I$$



$$\frac{\overline{h}C}{2e}\frac{d^2\phi}{dt^2}+\frac{\overline{h}}{2eR_N}\frac{d\phi}{dt}+I_c\sin\phi+I_F=I$$

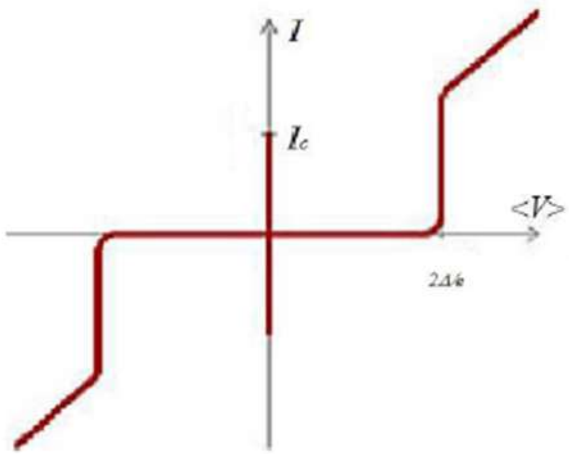
$$\beta_C\ddot{\phi}+\dot{\phi}+\sin\phi=i+i_F.$$

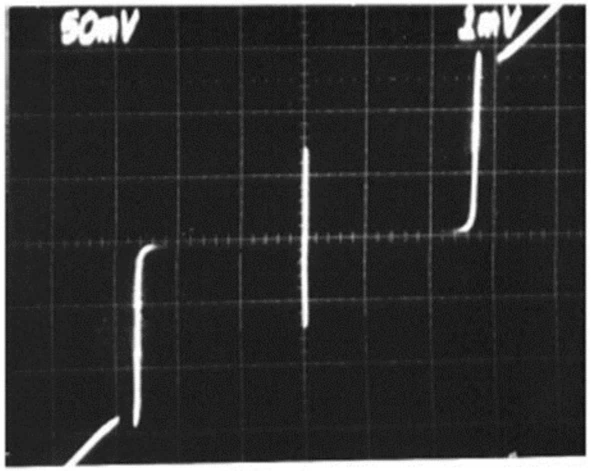
$$\beta_C=\frac{2\pi I_c R_N^2 C}{\Phi_0}$$

$$\frac{\Phi_0}{2\pi I_c R_N}$$

IV curve of underdamped JJ

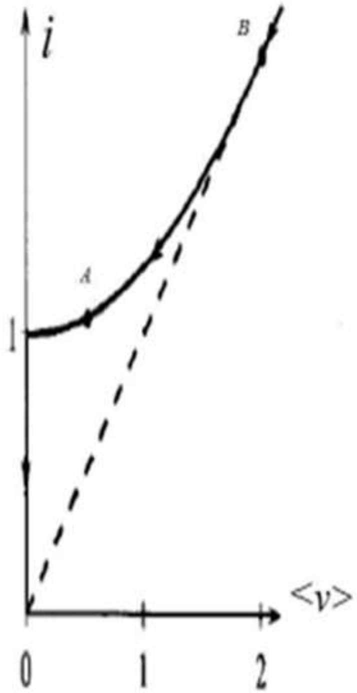
$$\beta_c = \frac{2e}{\hbar} I_c R_N^2 C >> 1$$

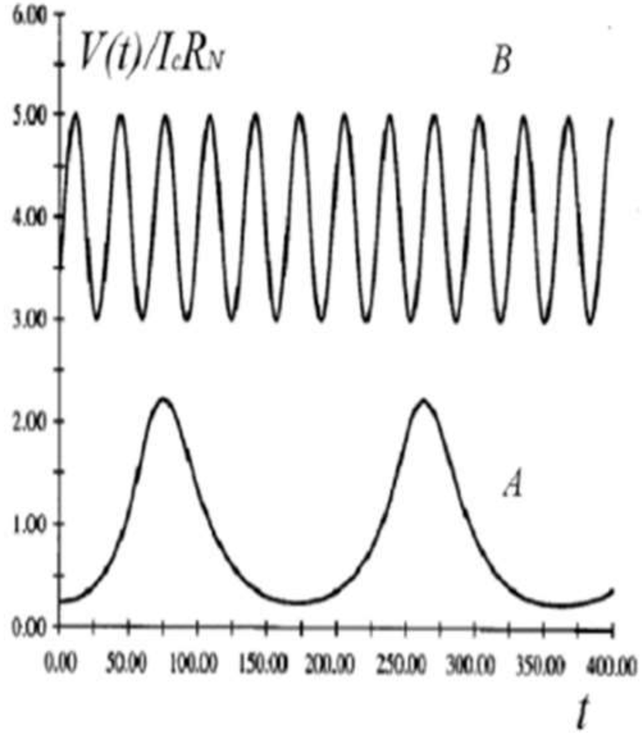




Latching technology on tunnel junctions: IBM Project 1980 years : ns

IV curve of overdamped JJ $\beta_C = \frac{2e}{\hbar} I_c R_N^2 C << 1$

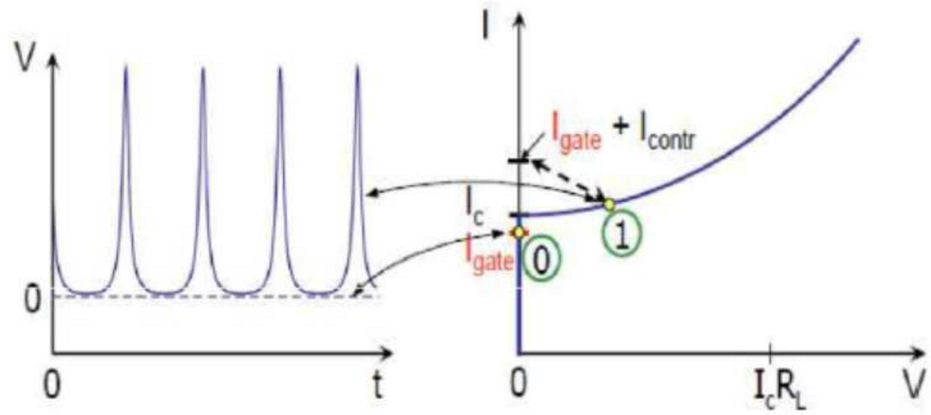




Time resolution at the level 1.2 ps

RSFQ Josephson logic

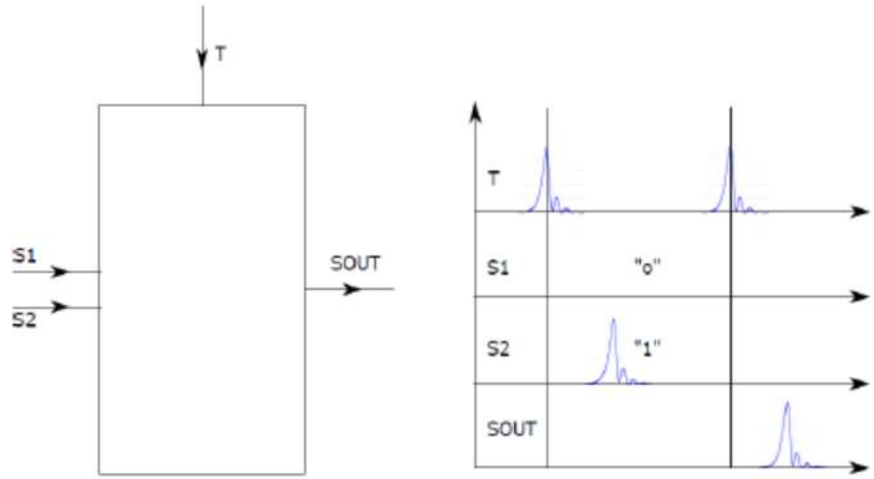
Josephson nonlatching technology on overdamped JJ



Suggested by Likharev, et al 1985,
Moscow State University

Realization by Likharev, et al 1999,
Stony Brook University

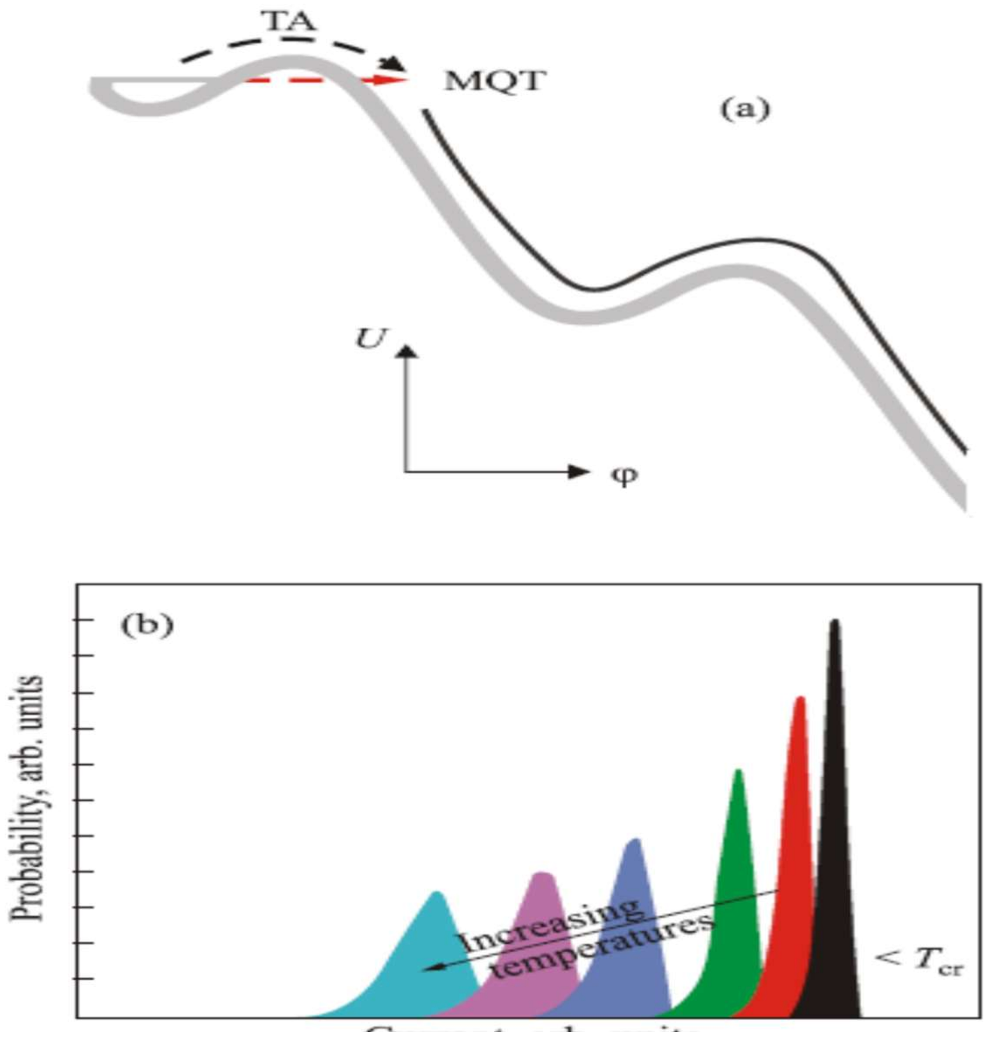
Operating frequency: 775 GHz !!!



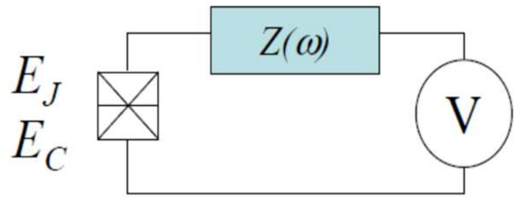
Thermal activation and MQT in JJ

Uncertainty relation for a superconductor: $\Delta n \cdot \Delta \varphi \geq 1$

Charging energy $E_C = \dfrac{e^2}{2C_J}$ Josephson energy $E_J = \dfrac{I_c \Phi_0}{2\pi}$



Quantum Fluctuations



$Z << R_Q = h/4e^2 = 6.45\ \mathrm{k}\Omega$

Josephson effect + quantum fluctuations of the phase

Perturbation theory, $E_J/E_C << 1$

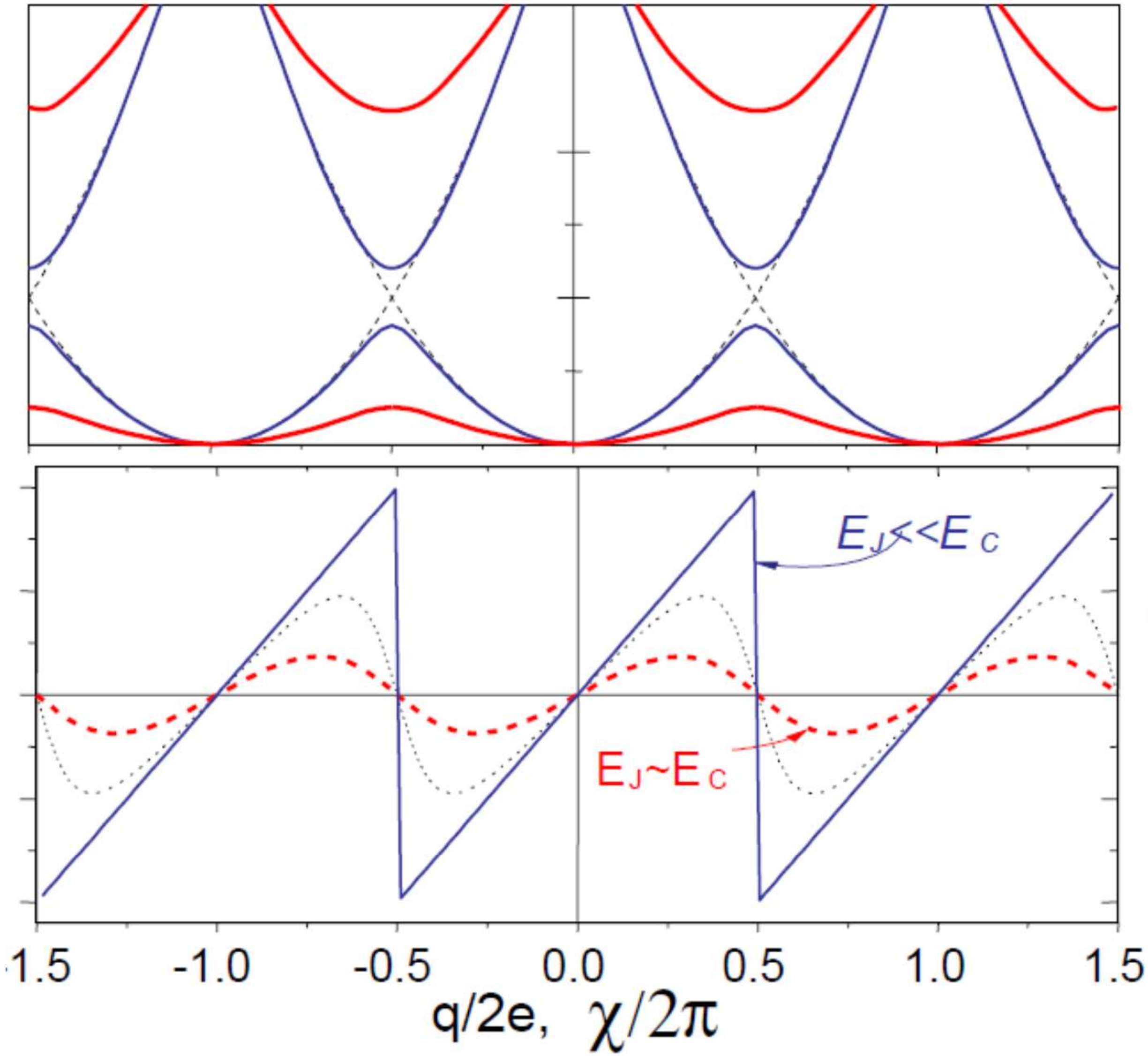
$Z >> R_Q = h/4e^2 = 6.45\ \mathrm{k}\Omega$

Coulomb blockade + charge fluctuations (uncorrelated single C.P.tunneling events)

Perturbation theory $E_J/E_C << (R_Q/Z)^{1/2}$

Quasi Charge description of Josephson Junctio

Averin, Likharev and Zorin 1985



$$H = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\phi}{\Phi}\right.$$

$$V = \frac{dE_0}{dq} = V_C \mathrm{saw}\chi$$

Critical Voltage: $V_C(E_J /$

dimensionless quasi-ch

$$\chi = \frac{2e}{2\pi} q$$

External source provides current bias

$$\boxed{I_{\mathrm{ext}} = \frac{2e}{2\pi} \dot{\chi}}$$

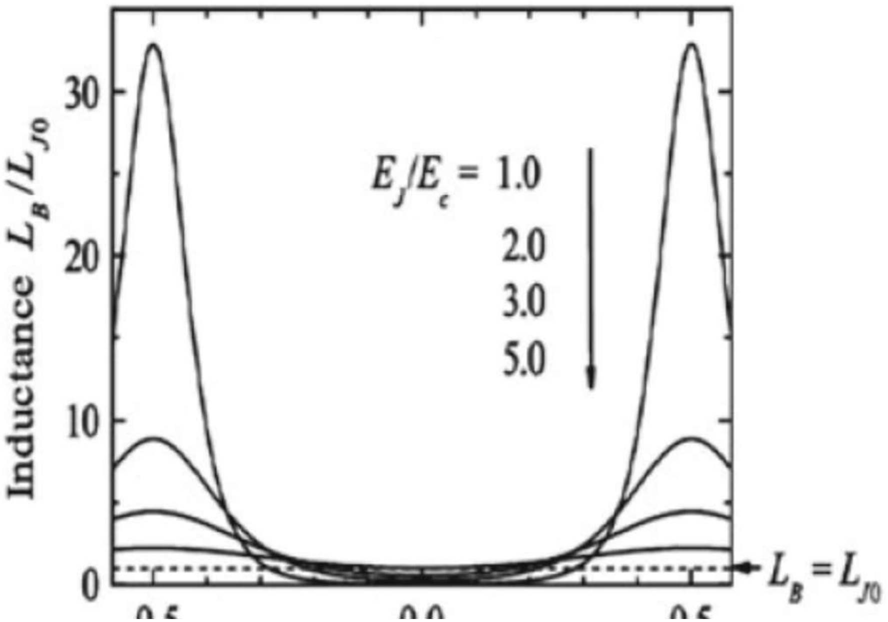
Basic equation of small JJ for quasicharge

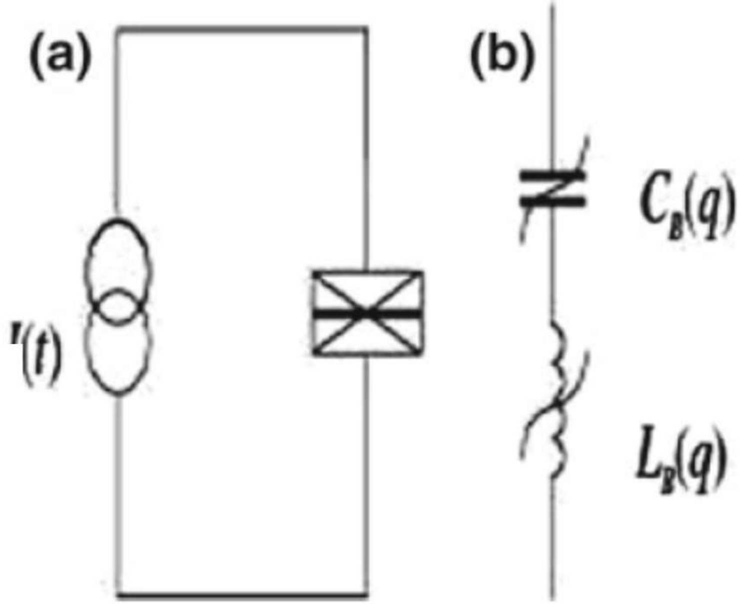
$$L_B(q)\frac{d^2q}{dt^2} + R\frac{dq}{dt} + V(q) = V_e;$$

$$V(q) = \frac{e}{C}\frac{\frac{q}{e} - \left(\frac{q}{e}\right)^3}{\sqrt{\left(\left(\frac{q}{e}\right)^2 - 1\right)^2 + \frac{\kappa^2}{4}}}$$

$$L_B = \frac{L_J}{(1+\xi^2)^2}, \quad L_J = \frac{\Phi_0}{2\pi I_c}; \ \xi < 1$$

$$C_B(q) = \left(\frac{d^2E_0(q)}{dq^2}\right)^{-1}$$

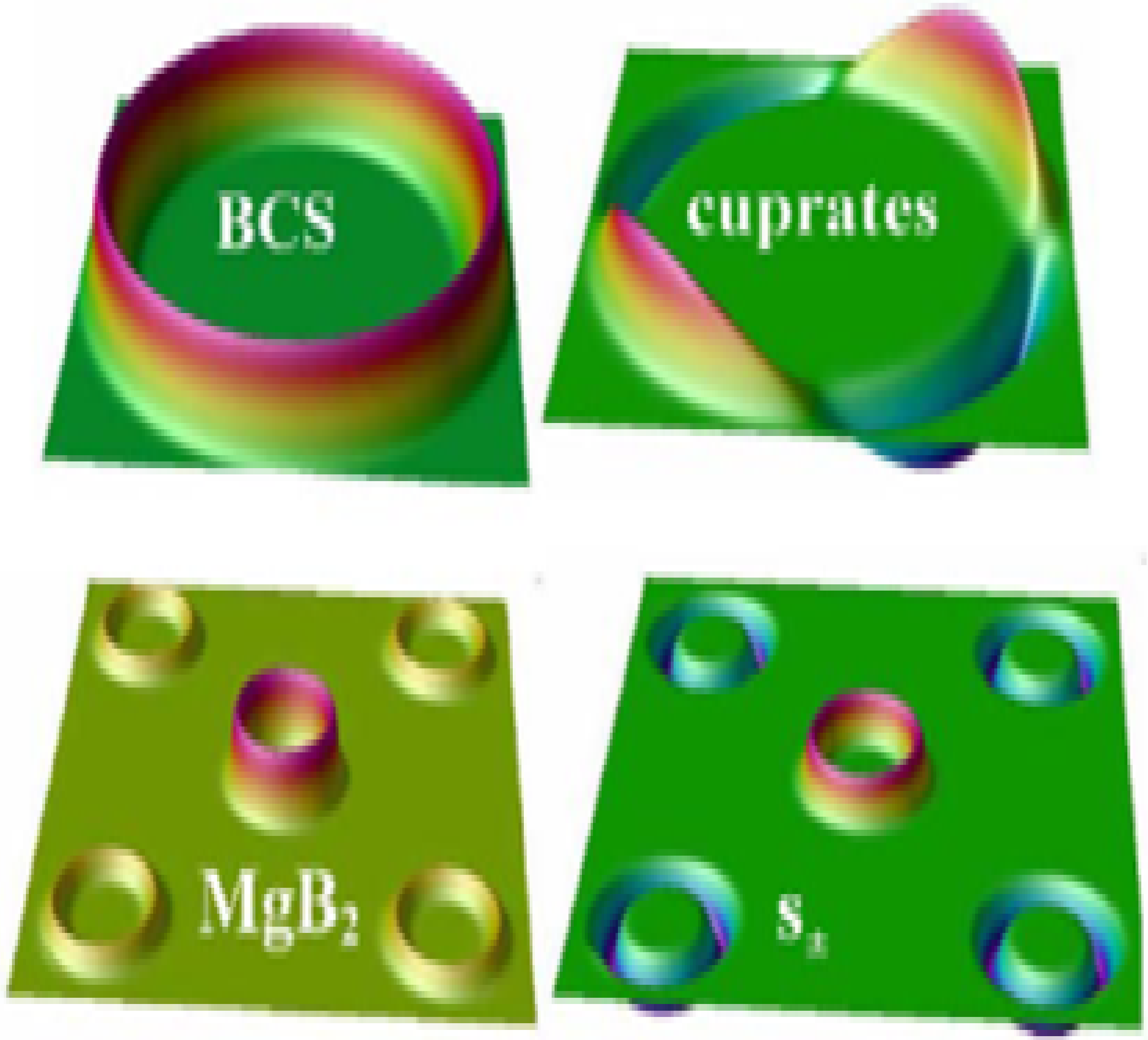




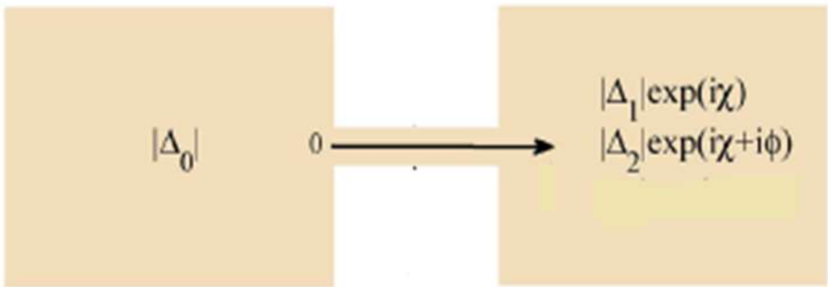
A. B. Zorin, *Phys. Rev. Lett.* 96 (2006) 167001.

Askerzade, MPLB, 2019

2. Order Parameter symmetry in different new compounds



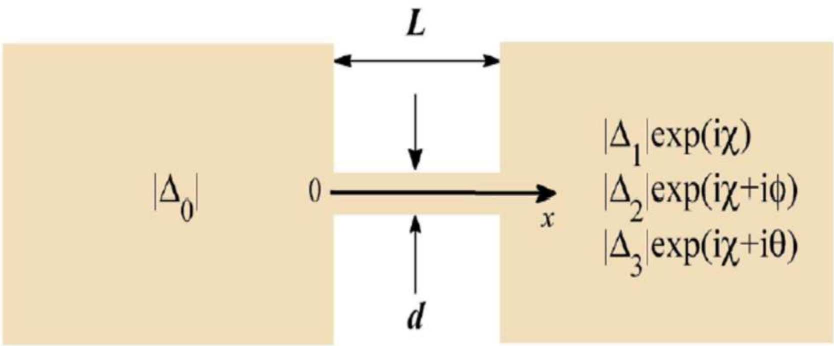
Josephson current in SB/many band superconductors



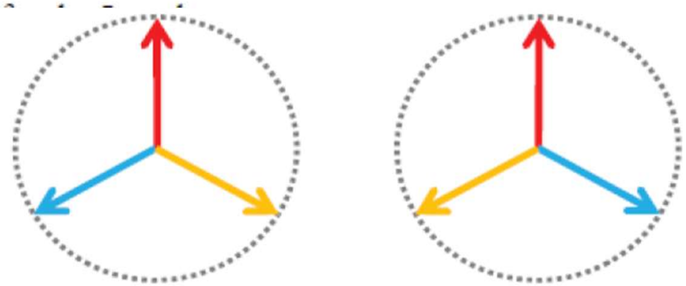
$$I = I_{c1}\sin\chi + I_{c2}\sin(\chi+\phi)$$

$$\phi = \begin{cases} \pi, \cdots if \ldots \varepsilon > 0 \\ 0, \cdots if \ldots \varepsilon < 0 \end{cases}$$

Askerzade, USC 2012



$$I = \frac{\pi|\Delta|^2}{4eT_cR_{N1}}\sin\chi + \frac{\pi|\Delta|^2}{4eT_cR_{N2}}\sin(\chi+\phi) + \frac{\pi|\Delta|^2}{4eT_cR_{N3}}\sin(\chi+\theta),$$



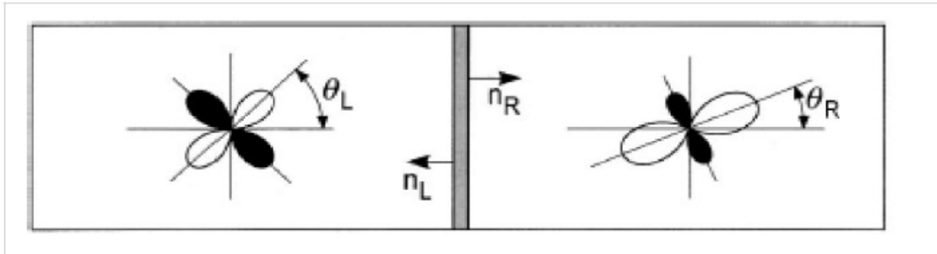
$\phi=\pi/3,\ \theta=-\pi/3;\quad \phi=-\pi/3,\ \theta=\pi/3$

$\phi=2\pi/3,\ \theta=-2\pi/3\quad \phi=-2\pi/3,\ \theta=2\pi/3$

Yerin,Omelyanchouk, LTP, 2014

D-wave HTSC: dwave-GL equations (Sigrist-Rice, 1993): SB/SB junctions

$$I(\phi) = \frac{4\pi ct}{\Phi_0}\chi_1(\mathbf{n}_1)\chi_1(\mathbf{n}_1)\,|\psi_1|\,|\psi_2|\sin\phi,$$



$$I(\phi) = A_s\cos(2\theta_L)\cos(2\theta_R)\sin\phi,$$

$$I(\phi) = A_s\cos 2(\theta_L+\theta_R)\sin\phi,$$

Tanaka-1997; Method of Green functions

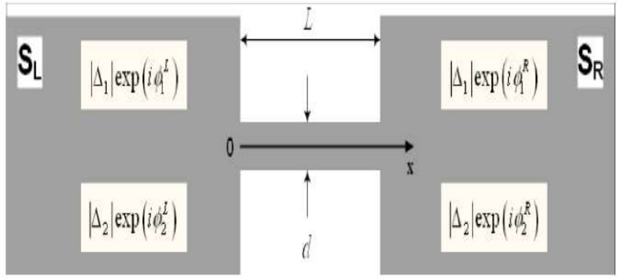
$$\bar{R}_N^{-1} = \int_{-\pi/2}^{\pi/2}\sigma_N\cos\theta d\theta;\quad \sigma_N = \frac{4Z_0^2}{(1-Z_0^2)\sinh^2(\lambda d_i)+4Z_0^2\cosh(\lambda d_i)}$$

$$\lambda = \sqrt{1-\kappa^2\cos^2\theta}\lambda_0 :\quad Z_0 = \frac{\kappa\cos\theta}{\sqrt{1-\kappa^2\cos^2\theta}}$$

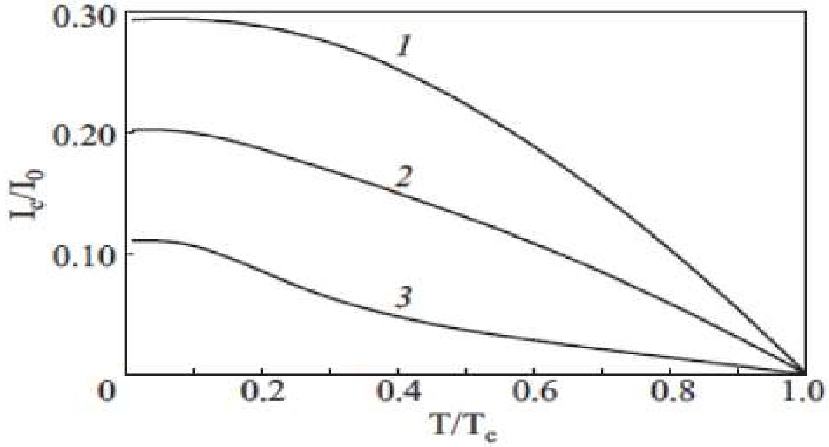
$$I(\phi) = \sum_{n>1}(I_n\sin n\phi + J_n\cos n\phi)$$

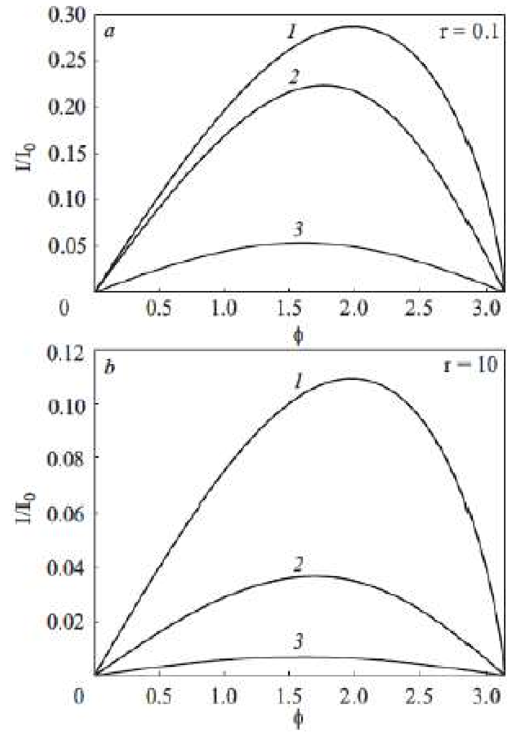
Two-band SC based JJ: Yerin-Omelyanchouk (2011)

Green function method: generalization of Baratoff results



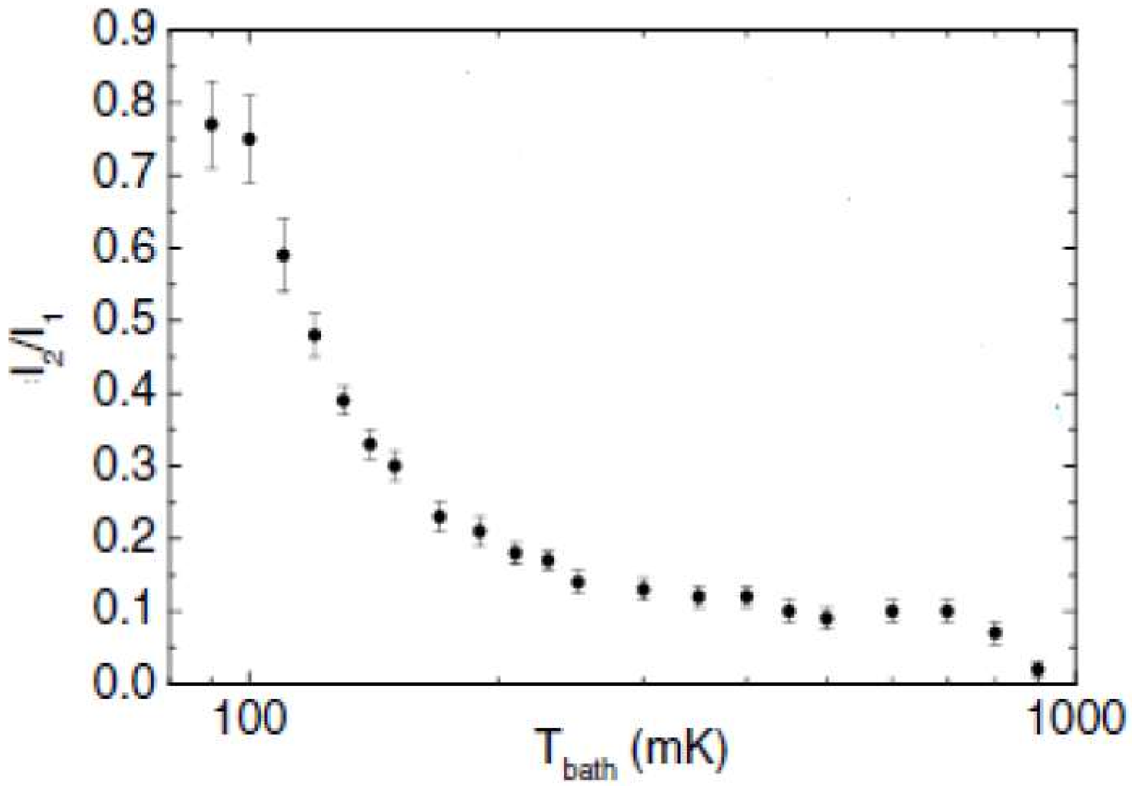
$$I(\phi) = \left\{ \begin{array}{l} \frac{4\pi T}{eR_{N1}} \sum_{\omega>0} \frac{\Delta_1 \cos(\phi/2)}{\sqrt{\omega^2+\Delta_1^2\cos^2(\phi/2)}} atrc\tan \frac{\Delta_1 \sin(\phi/2)}{\sqrt{\omega^2+\Delta_1^2\cos^2(\phi/2)}} + \\ + \frac{4\pi T}{eR_{N2}} \sum_{\omega>0} \frac{\Delta_2 \cos(\phi/2)}{\sqrt{\omega^2+\Delta_2^2\cos^2(\phi/2)}} atrc\tan \frac{\Delta_2 \sin(\phi/2)}{\sqrt{\omega^2+\Delta_2^2\cos^2(\phi/2)}} \end{array} \right\}$$





JJ Anharmonic CPR : YBCO grain boundary

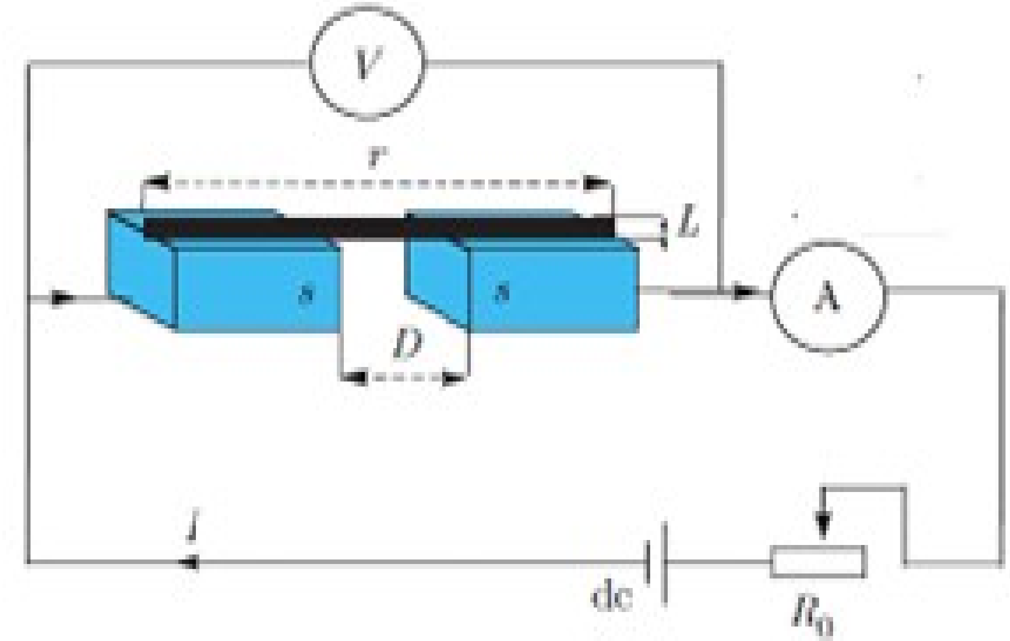
$$I = I_{c0}(\sin\phi + \alpha\sin(2\phi))$$



Bauch T. et al , Phys.Rev. Lett, 94,087003(2005)

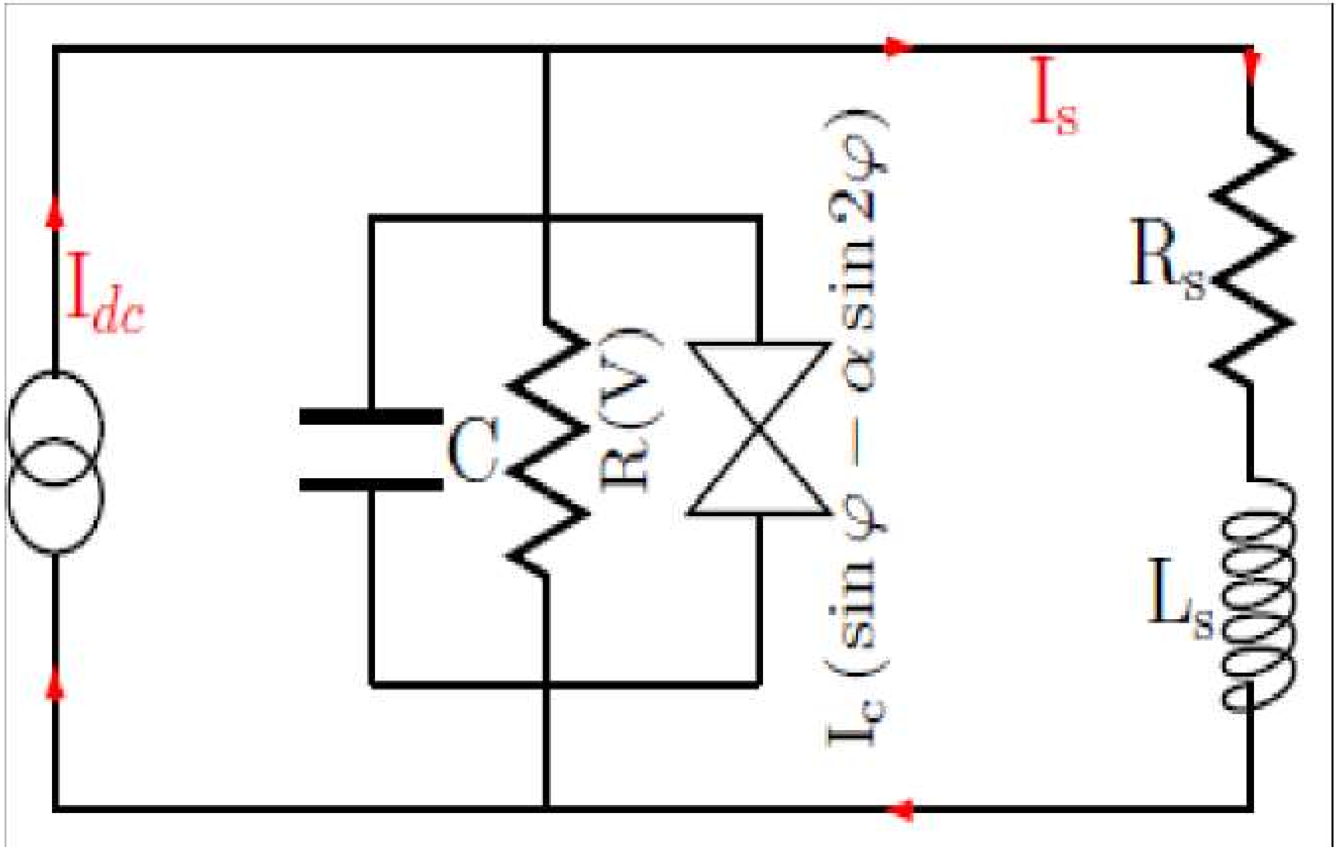
Majorana fractional term in CPR (p-wave SC)

$$I = I_{c0}(\sin\phi + m\sin(\phi/2))$$



JJ with nontrivial barrier reveal CPR with fractional term

Unconventional CPR JJ: modelling

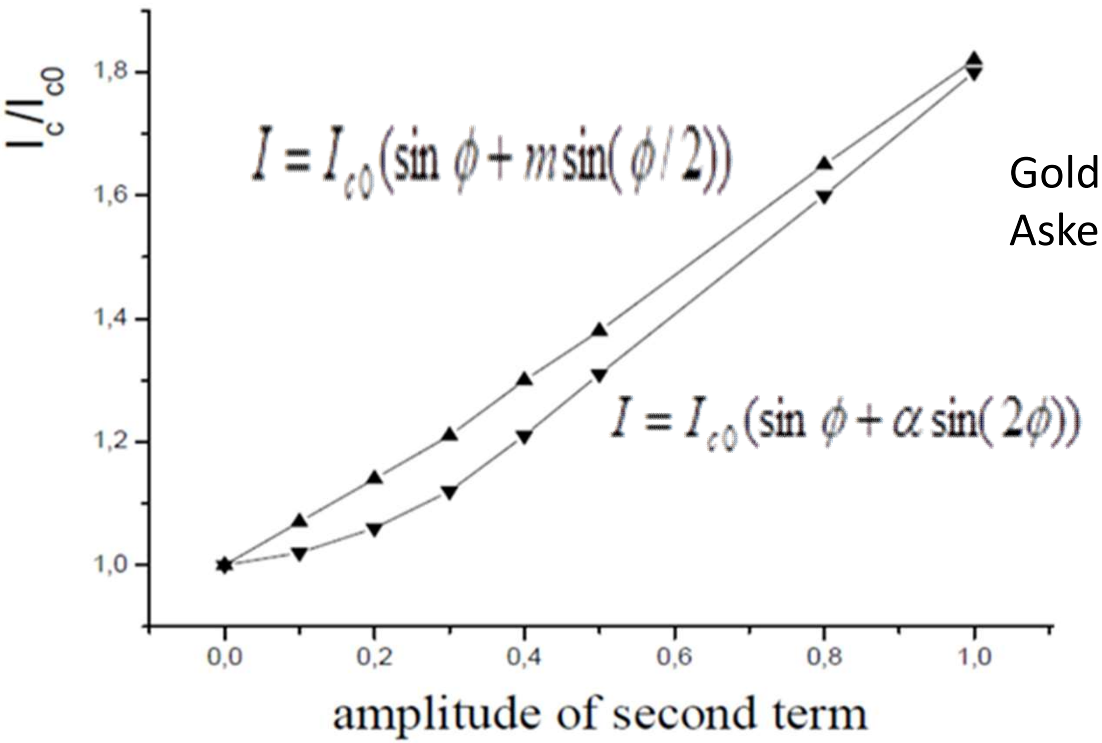


Canturk M., I.N.Askerzade I.N., IEEE Applied Superconductivity, 22,1400106 (2012)

JJ dynamics equation in general case

$$\begin{aligned}\dot{\phi} &= v,\\ \beta_C \dot{v} + g(v)v + f(\phi) + i_s &= i,\\ \beta_L \dot{i}_s + i_s &= v,\end{aligned}$$

$$R(V) = \begin{cases} R_N & \text{if } |V| > V_g \\ R_{sg} & \text{if } |V| \leq V_g \end{cases}$$



Goldobin E. et al 2007 Phys. Rev. B
Askerzade, JSNM, 2019

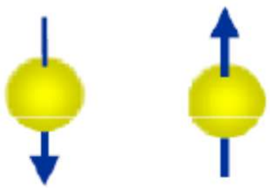
3. Superconducting qubits

Important steps to quantum computing

- Shor algorithm repulse Crush-Turing thesis: changing of computation power do not change complexity of problem
- Atomic scale resolution nanotechnolgy
- Low temperature experimental technique (nK)
- Development low dimensional physics

Qubits

Quantum states



$|0\rangle$ $|1\rangle$

two basis states

Wavefunction

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

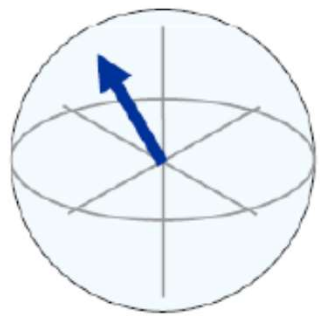
α and β are complex numbers

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In principle, any two-level system which acts quantum mechanically can be used as a qubit.

The Bloch sphere

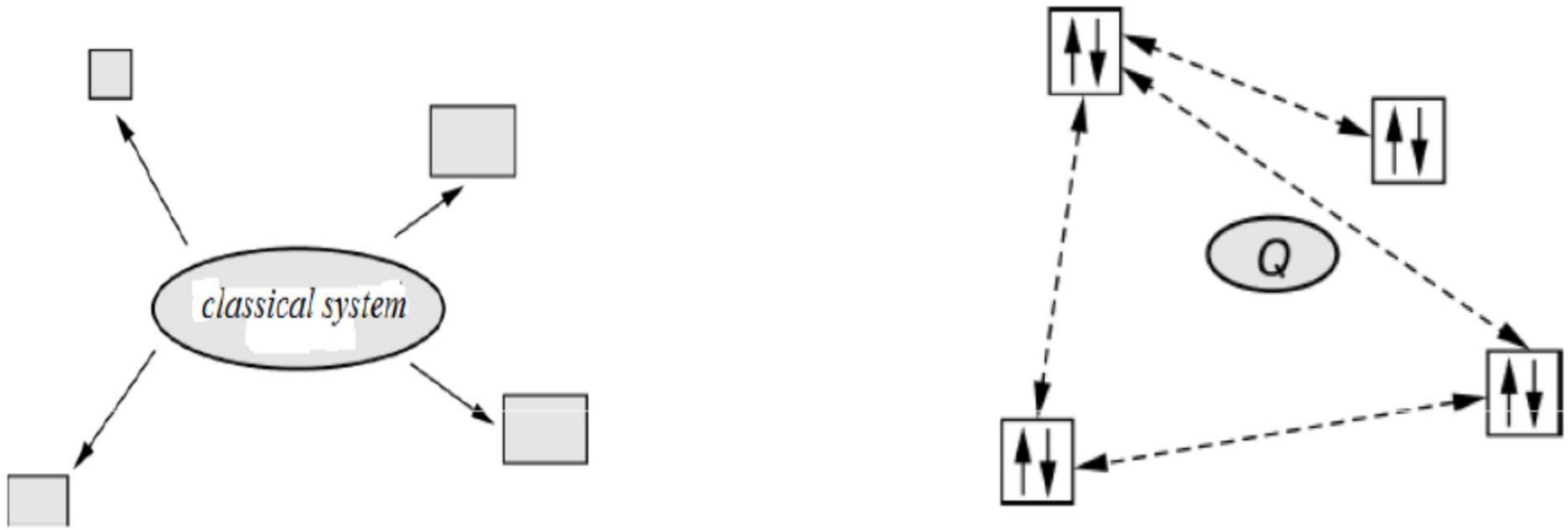
$|1\rangle$



$|0\rangle$

$$|\alpha|^2 + |\beta|^2 = 1$$

Classical and quantum systems



Changing of state of only one part (qubit) changes entire superposition, which lead 2^n –fold quantum parallelism of computation

Entangled states

Entangled states for two qubit systems

$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

No Entangled states: $\alpha\delta - \beta\gamma = 0$

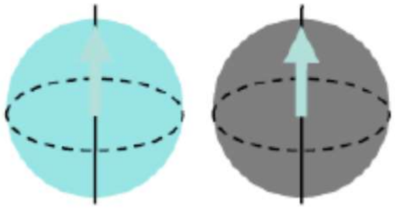
Factorization: $|\Psi\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$

$$\alpha\delta - \beta\gamma \neq 0$$

No factorization, entangled states

Entanglement of states

Some two-qubit states can be obtained as a **product of single-qubit states**, e.q.



$$|\Psi\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

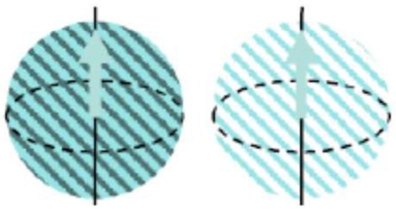
Entangled state can NOT be obtained as a product of single-qubit states

Examples: $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ Bell state

measurement result

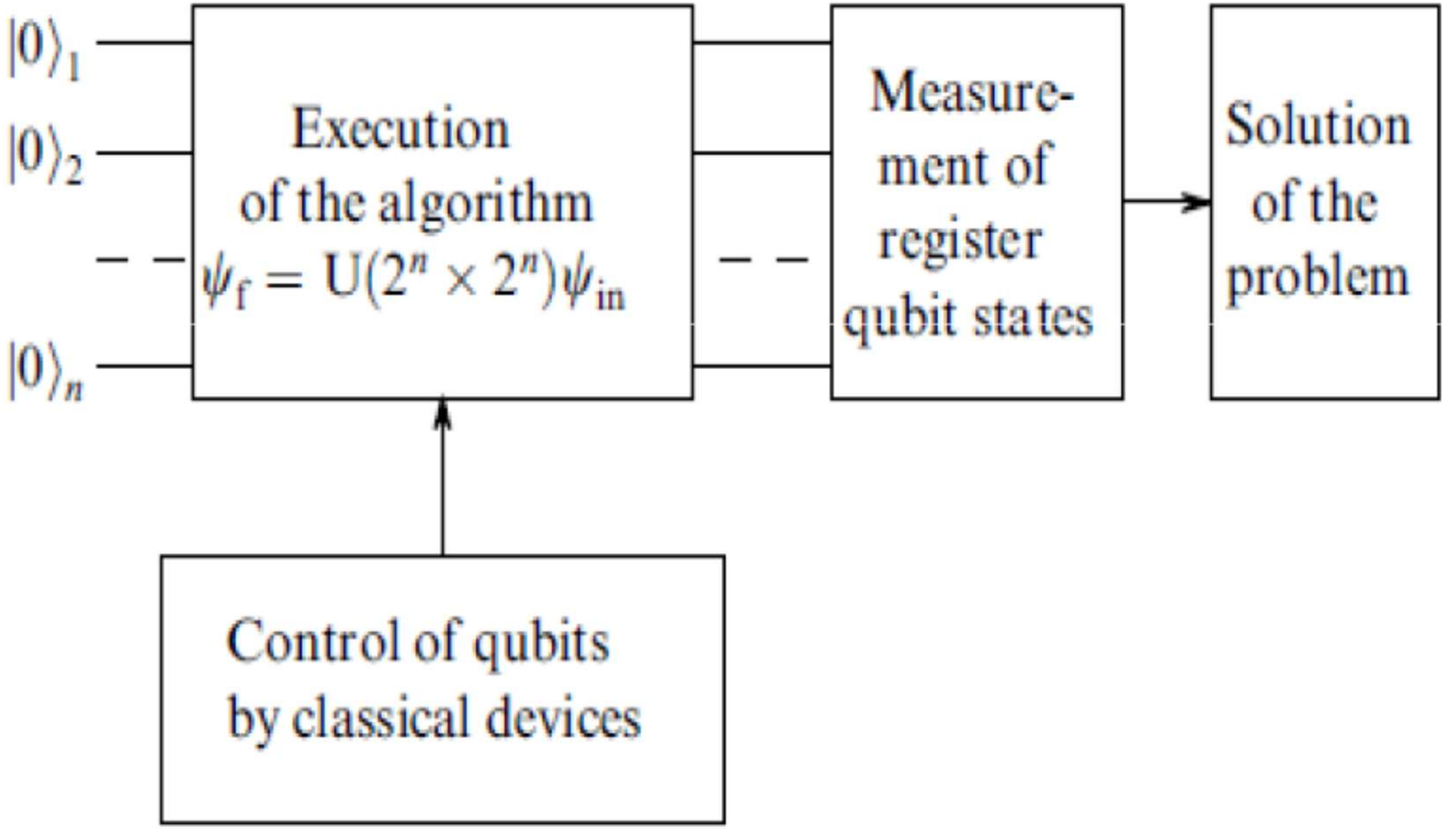
$|00\rangle$ with probability ½

$|11\rangle$ with probability ½



$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$ EPR (Einstein–Podolsky–Rosen) pair

Schematic of quantum computer



Possible qubits

- Microscopic
  - ensemble of spins (NMR)
  - ions in electromagnetic traps
  - neutral atoms
  - photons in cavities
  - single-atom defects (e.g. NV centers)

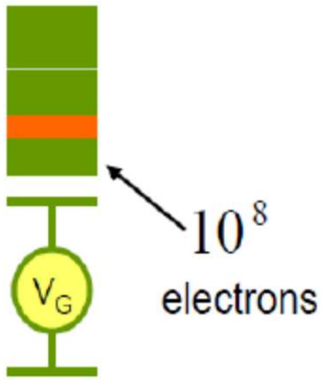
- Macroscopic or mesoscopic
  - spins in solid-state nanodevices
  - electrons on superfluid helium
  - charge, phase or flux in superconductors

Uncertainty relation for a superconductor: $\Delta n \cdot \Delta \varphi \geq 1$

Charging energy $E_C = \dfrac{e^2}{2C_J}$

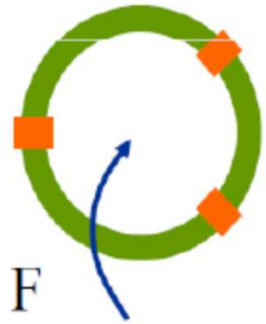
Josephson energy $E_J = \dfrac{I_c \Phi_0}{2\pi}$

charge qubit



$E_C >> E_J$

flux qubit



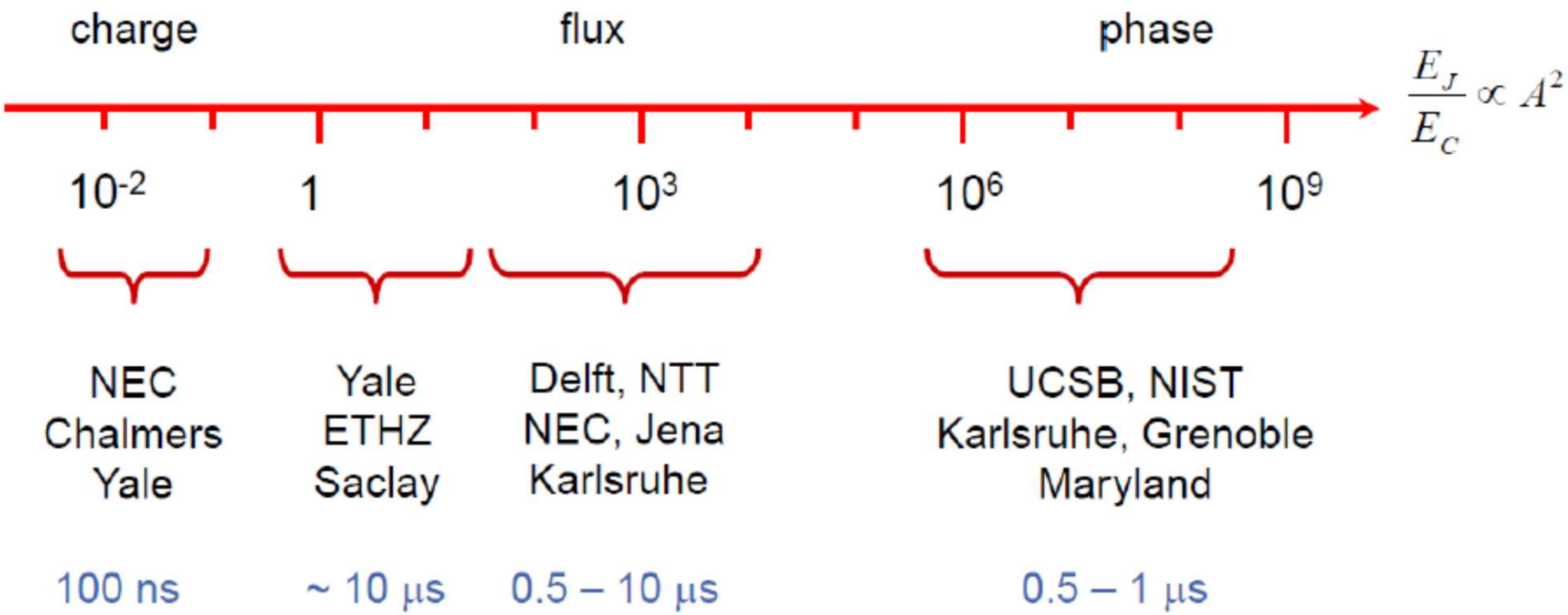
$E_C << E_J$

- Makhlin et al., Nature **398**, 305 (1999)
- Nakamura et al., Nature **398**, 786 (1999)

- Friedman et al., Nature **406**, 43 (2000)
- van der Wahl et al., Science **290**, 773 (2000)

Josephson qubits: energy scale

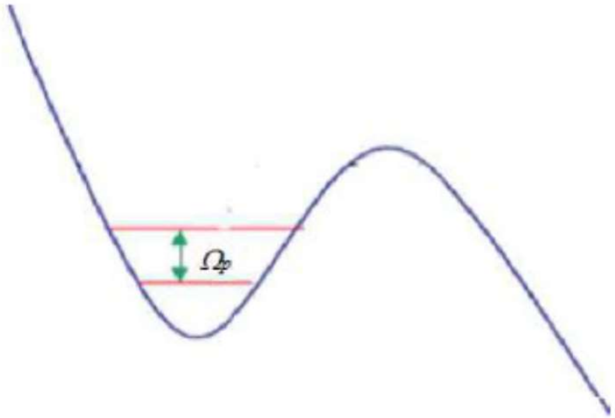
for a chosen J_c, the ratio E_J/E_C depends on the junction area A



Josephson phase qubit

Hamiltonian of phase qubit

$$H = -E_c \frac{\partial^2}{\partial \phi^2} + E_J \left\{ i_b \phi + \cos\phi \right\}$$



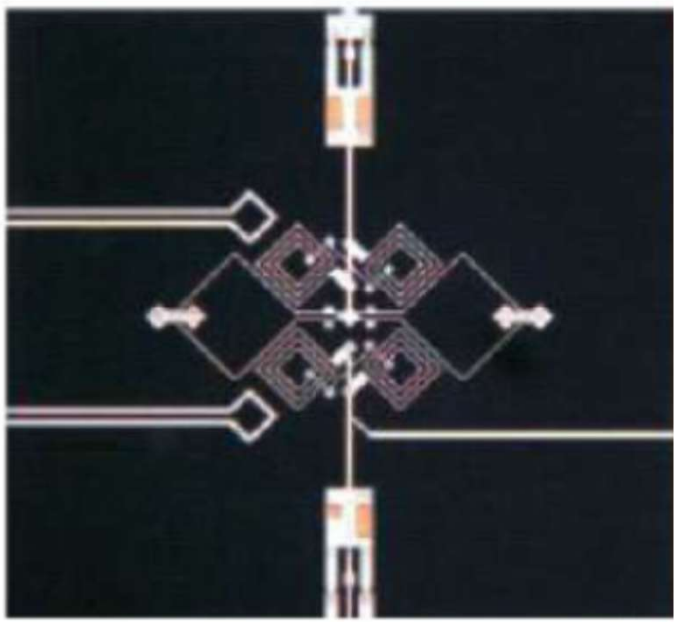
Quantum oscillator spectrum

$$E_{n0} = \hbar \Omega_p (n_0 + 1/2)$$

Plasman frequency

$$\Omega_p = \omega_J (1 - I_b/I_c)^{1/4}$$

$$\omega_J = I_c/2e.$$



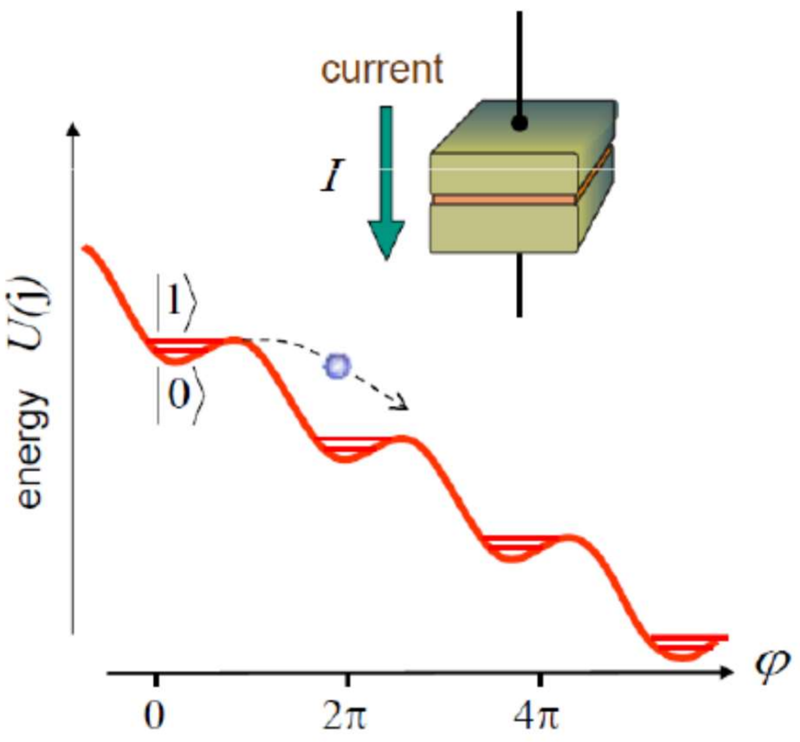
First realization of Josephson phase qubit:

J.M. Martinis, et al, Physical Review Letters **89**, 117901 (2002)

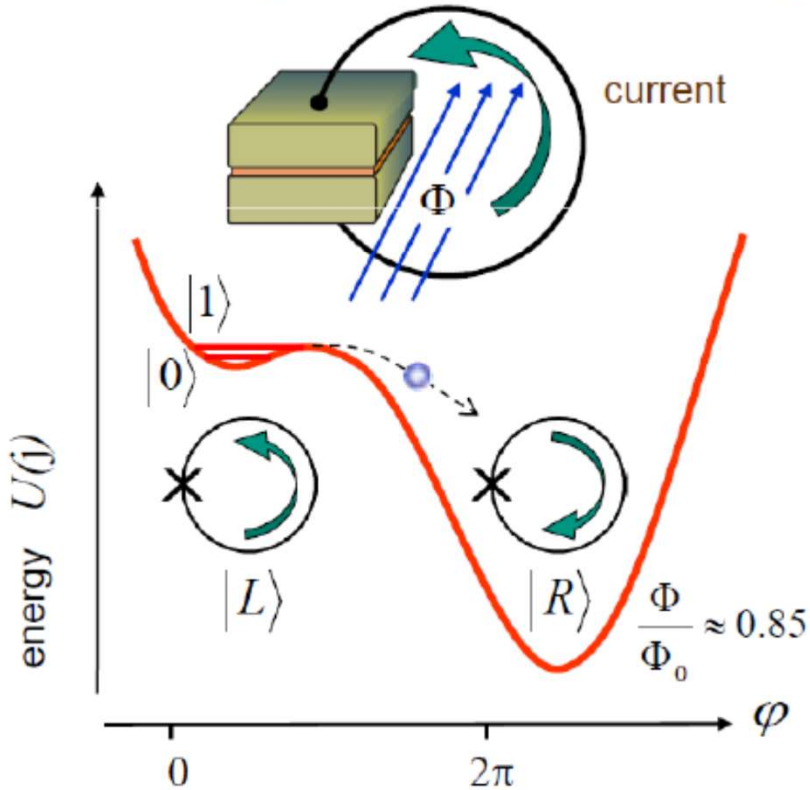
Coherence time is short: < mks

Josephson phase and flux qubits

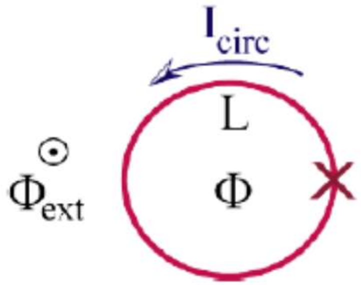
$$U(\varphi) = \frac{I_c \Phi_0}{2\pi}\left(-\frac{I}{I_c}\varphi - \cos\varphi\right)$$

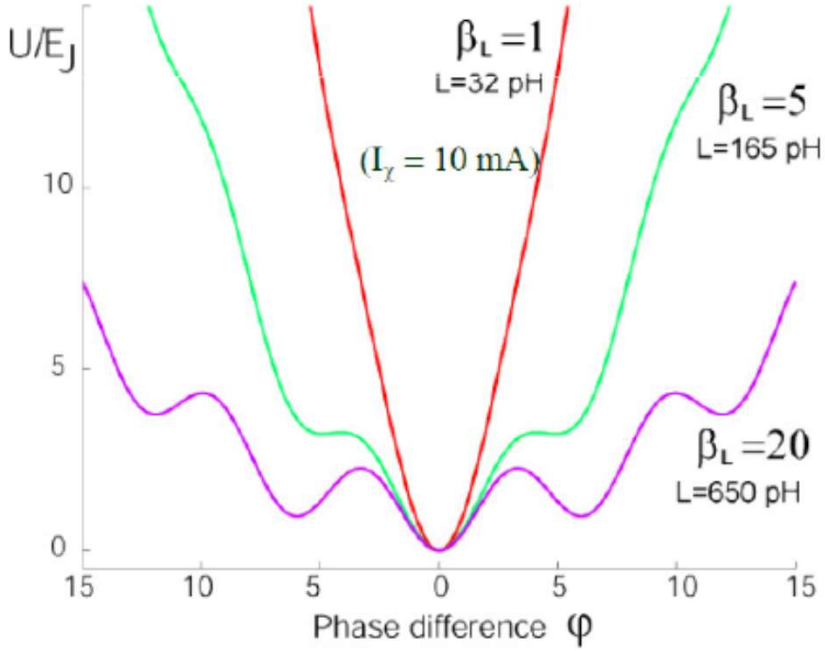


$$U(\varphi) = \frac{I_c \Phi_0}{2\pi}\left[\frac{1}{2\beta_L}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2 - \cos\varphi\right]$$



Single JJ flux qubit





potential energy:

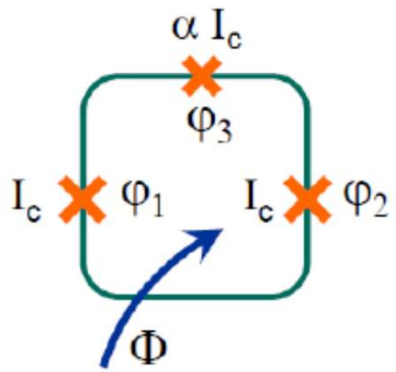
$$U(\varphi) = \frac{I_c \Phi_0}{2\pi}\left[\underbrace{\frac{1}{2\beta_L}\left(\varphi - 2\pi\frac{\Phi}{\Phi_0}\right)^2}_{\text{magnetic energy}} \underbrace{- \cos\varphi}_{\text{junction energy}}\right]$$

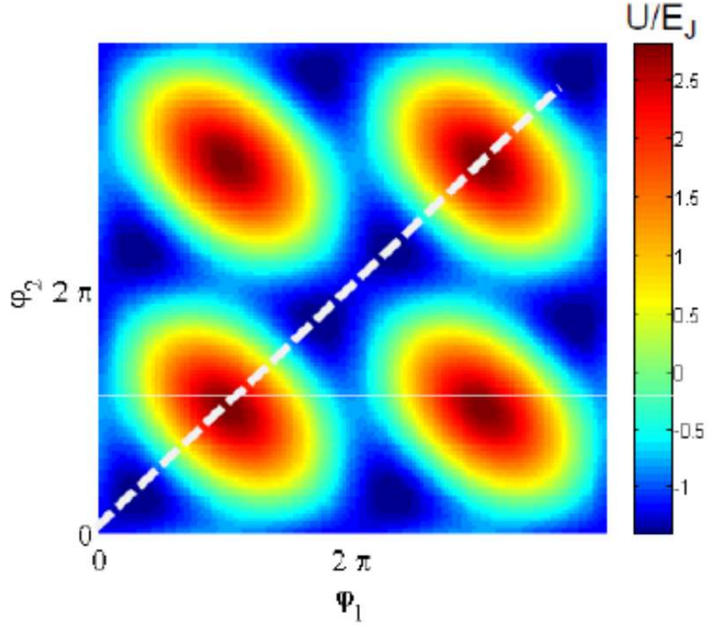
$$\beta_L \equiv \frac{2\pi L I_c}{\Phi_0}$$

Due to large loop inductance is sensitive t flux noise, coherence time is short about 20 ns

Three JJ flux qubit

$$L_J = \Phi_0 / I_c$$



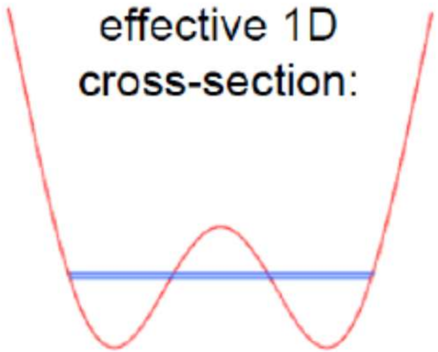


flux quantization:

$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi \frac{\Phi}{\Phi_0} = 2\pi\, n$$

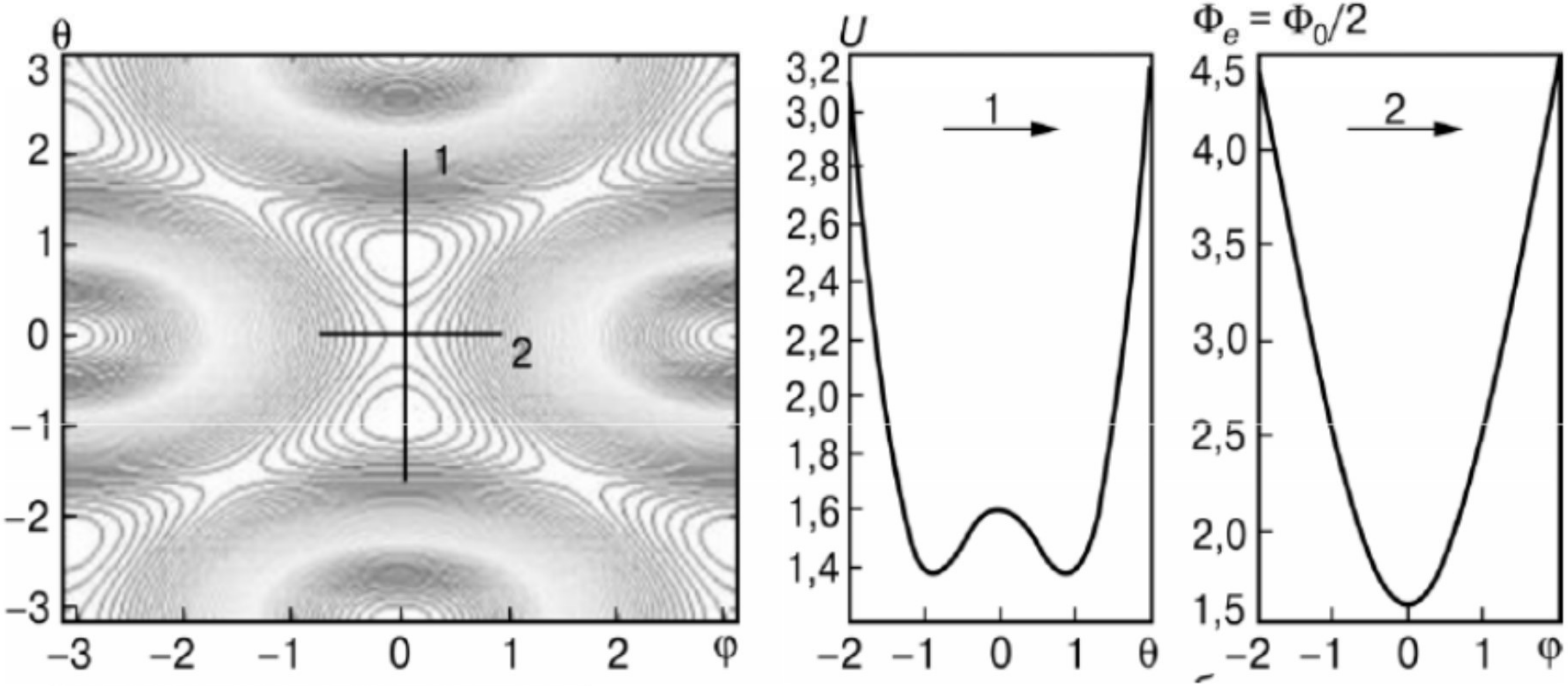
effective 2D potential:

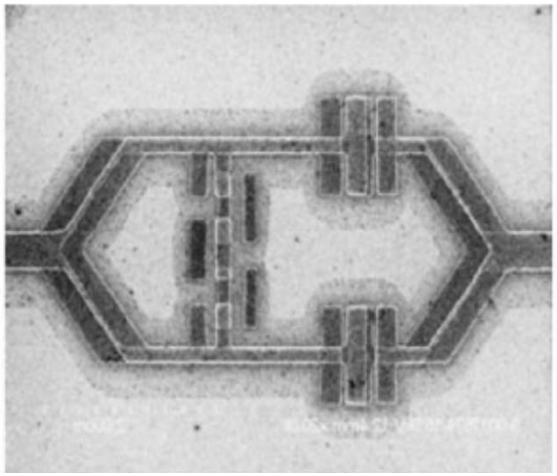
$$\frac{U}{E_J} = \cos\varphi_1 + \cos\varphi_2 + \alpha\cos\left(-\varphi_1 - \varphi_2 - 2\pi \frac{\Phi_{ext}}{\Phi_0}\right)$$



Mooij et al. Science 285, 1036 (1999)
Van der Wal et al. Science 290,1140 (2000)

Effective cross-section of potential in three JJ qubit

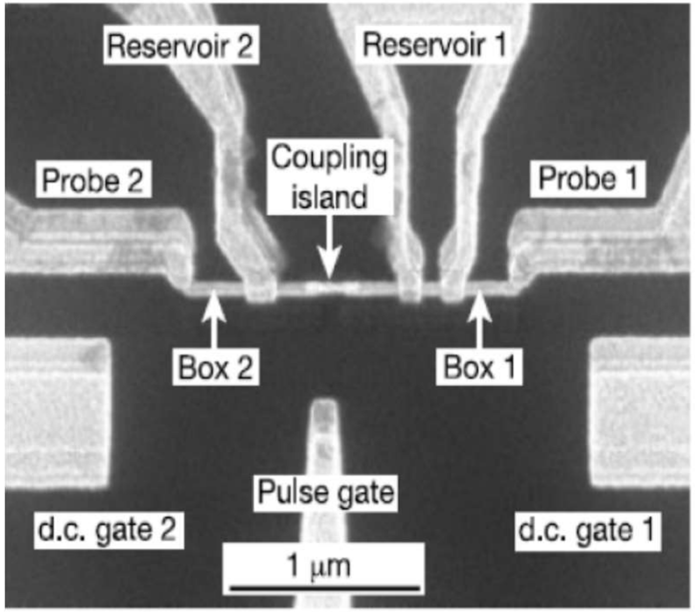


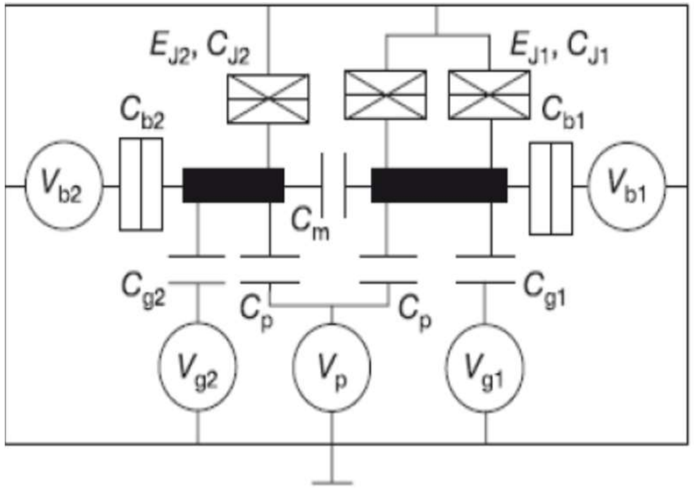


Coherence time is not good < mks

Mooij et al, Science, 285, 1036 (1999)

Two coupled charge qubits: 2003



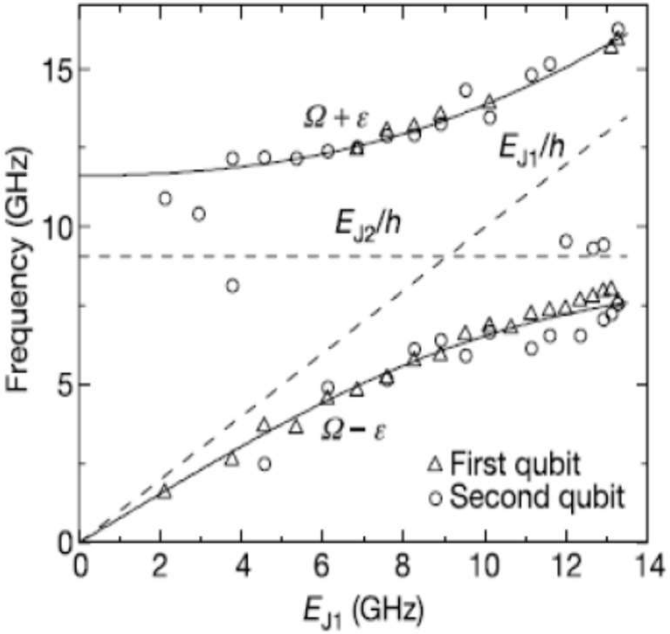


Quantum oscillations in two coupled charge qubits

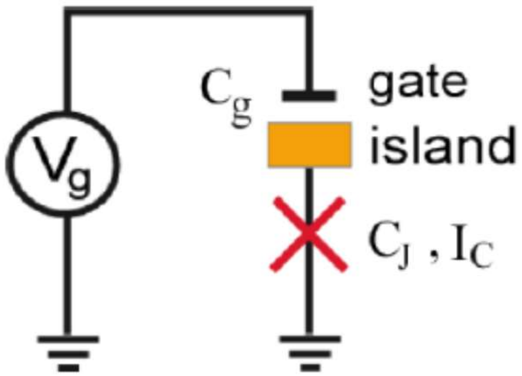
Yu. A. Pashkin*†, T. Yamamoto*‡, O. Astafiev*, Y. Nakamura*‡, D. V. Averin§ & J. S. Tsai*‡

NATURE | VOL 421 | 20 FEBRUARY 2003

** The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan*
‡ NEC Fundamental Research Laboratories, Tsukuba, Ibaraki 305-8501, Japan
§ Department of Physics and Astronomy, SUNY Stony Brook, New York 11794-3800, USA



Charge qubit with JJ



- charging energy: $E_{\text{charge}} = \dfrac{Q^2}{2C} = N^2 \underbrace{\boxed{\dfrac{(2e)^2}{2(C_g + C_J)}}}_{E_C}$

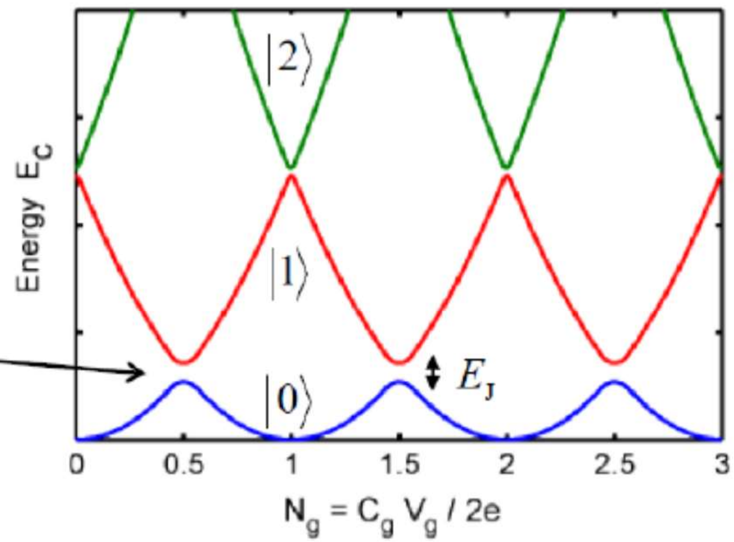
  with N the number of excess CPs on the island

- Josephson energy: $E_J = \dfrac{I_c \Phi_0}{2\pi}$

➡ Hamiltonian:

$$\boxed{\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\varphi}}$$

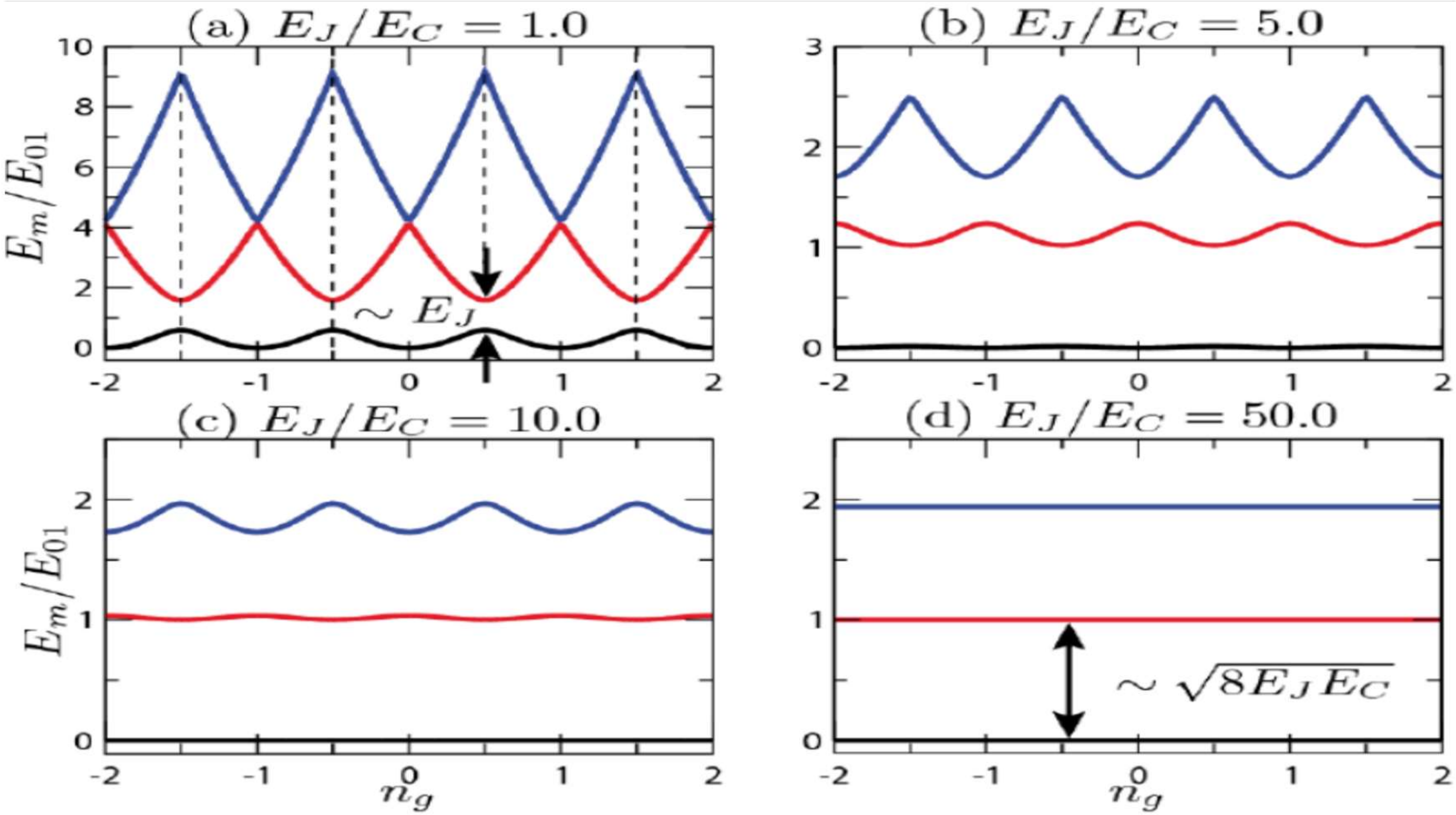
- The Josephson coupling lifts the charge degeneracy.
- Energy bands appear in the periodic potential (similar to atom lattice)



Transmon qubit

Charge qubits: sensitivity to charge fluctuations and low coherence time

For this purpose transmon qubits proposed by J. Koch et al

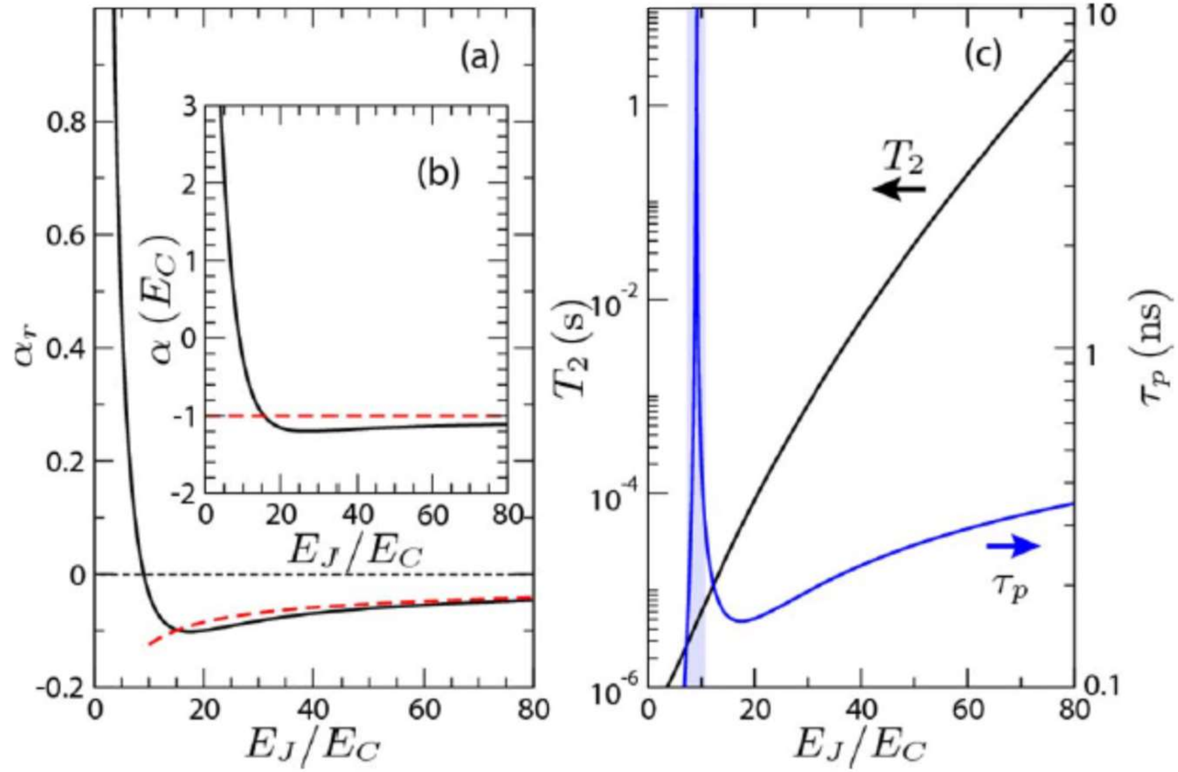


J. Koch et al., Phys. Rev. A **76**, 042319 (2007)

Transmon qubit

$$\alpha \equiv E_{12} - E_{01}, \quad \alpha_r \equiv \alpha / E_{01}$$

Anharmonicity of the transmon



J. Koch et al., Phys. Rev. A **76**, 042319 (2007)

Key element of several qubit projects !!!!!

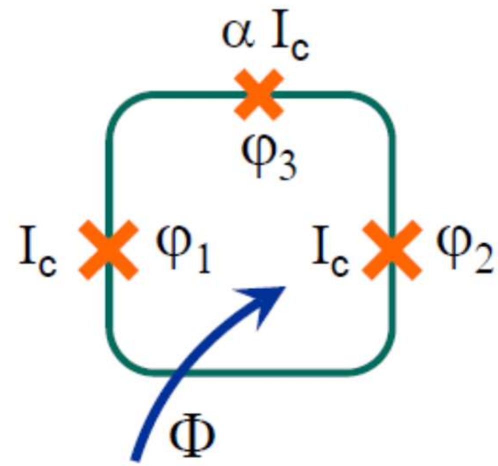
C-shunt flux qubit : Steffen et al, 2010PRL

C-shunt flux qubit is the three junction flux qubit shunted by a large capacitance

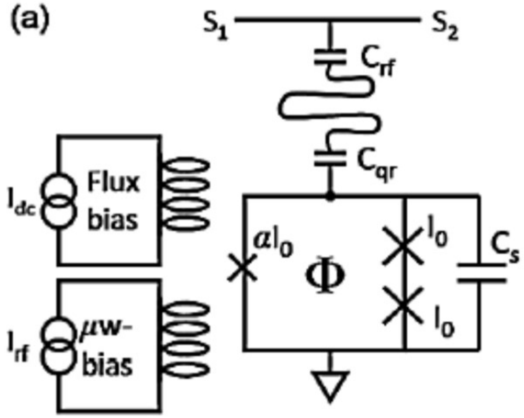
Potential has a single-well form in contrast to three junction qubits

C-shun flux qubit :

- Coherence time: is about 1.5 mks
- Strong anharmonicity
- High reprocibility



Three junction flux qubit



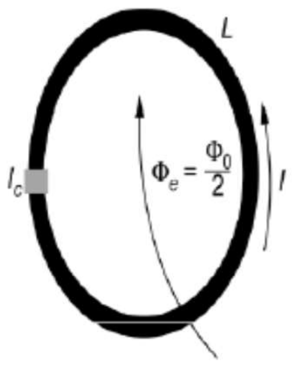
$$\alpha = 0.3$$

C-shunt flux qubit

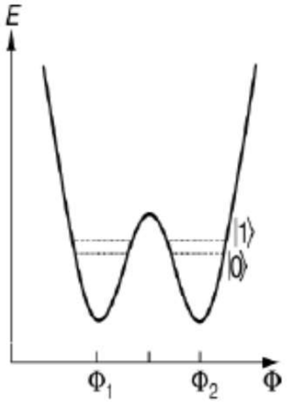
4. Qubits on JJ with anharmonic CPR

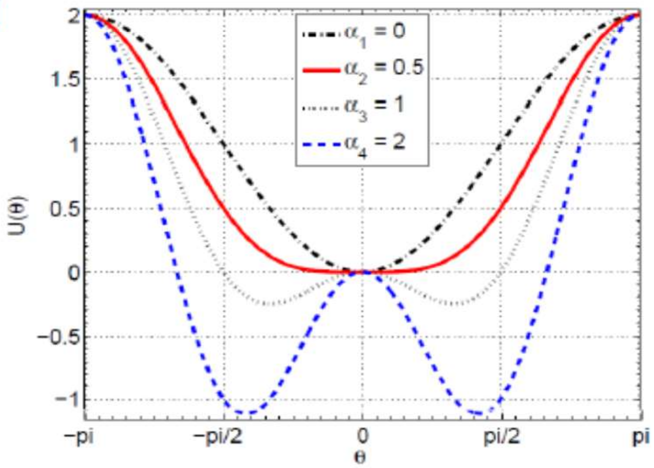
Silent flux qubit

$\phi_e = \pi$



Two-hump potential





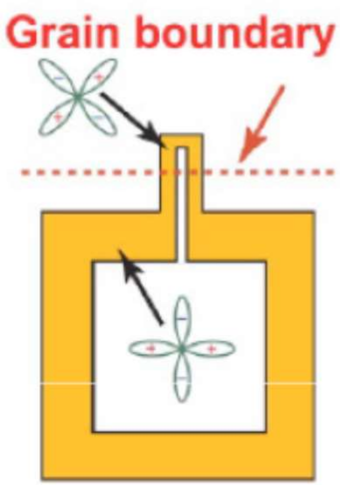
$\phi_e = 0$

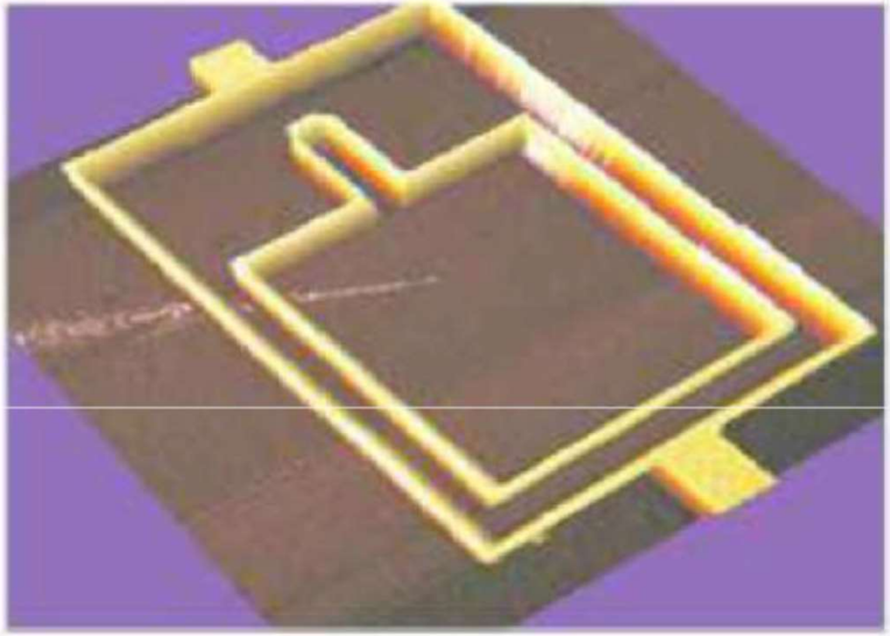
External flux fluctuations lead to decohering

High protection againts external magnetic field

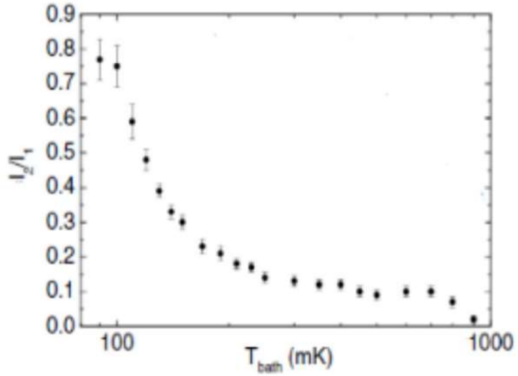
$$U(\phi,\phi_+) = -\frac{\Phi_0 I_{c1}}{2\pi}\left\{\cos\left(\frac{\phi}{2}+\phi_+\right) - \frac{\alpha_1}{2}\cos(\phi+2\phi_+)\right\}$$
$$-\frac{\Phi_0 I_{c2}}{2\pi}\left\{\cos\left(\frac{\phi}{2}-\phi_+\right) - \frac{\alpha_2}{2}\cos(\phi-2\phi_+)\right\},$$

Realization of silent qubit





YBCO grain boundary JJ



MHS Amin et al , PRB, 71(2005)

JJ phase qubit with anharmonic CPR

$$H = -E_c \frac{\partial^2}{\partial \phi^2} + E_j \left\{ i_b \phi + \cos\phi - \frac{\alpha}{2} \cos 2\phi \right\}$$

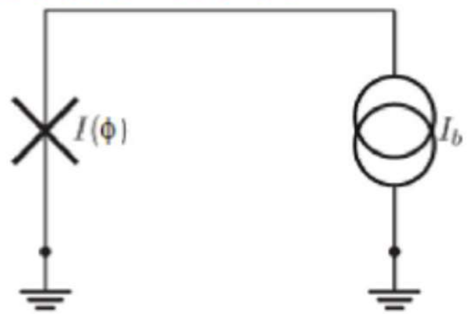
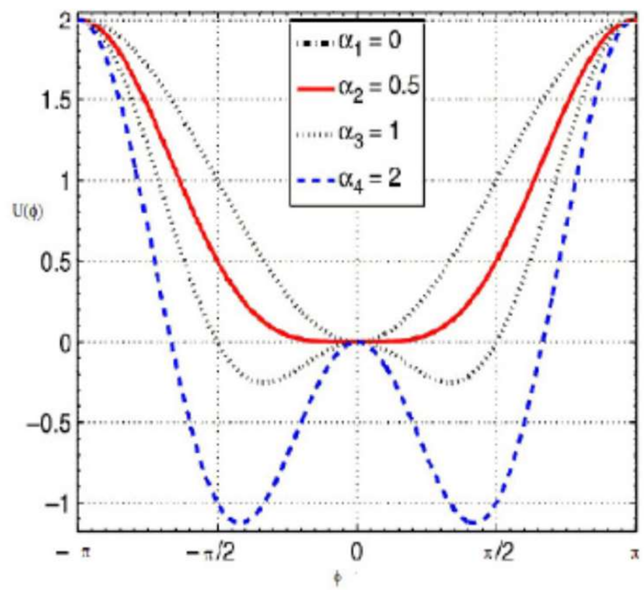


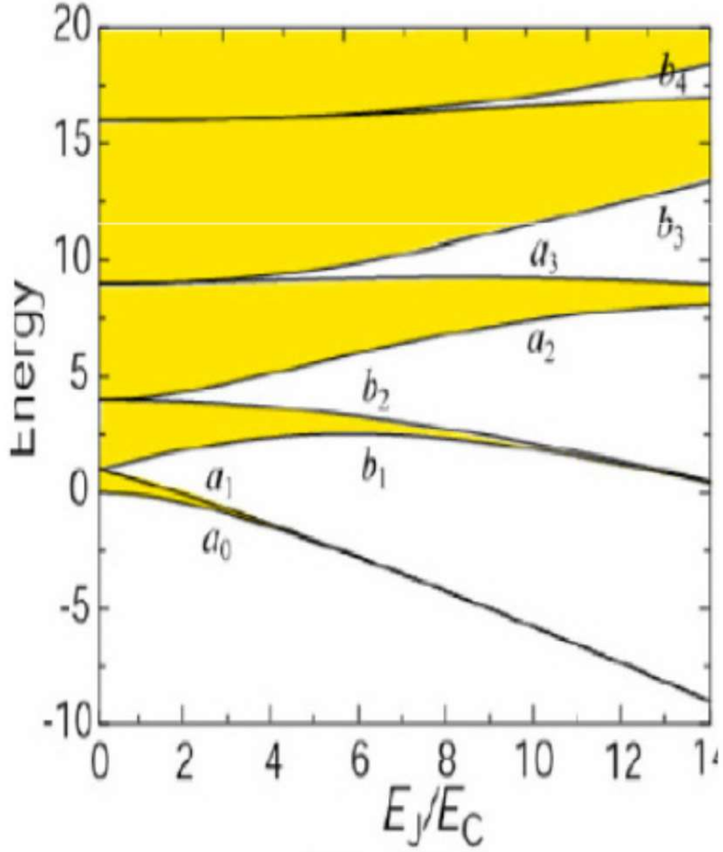
Figure 24:

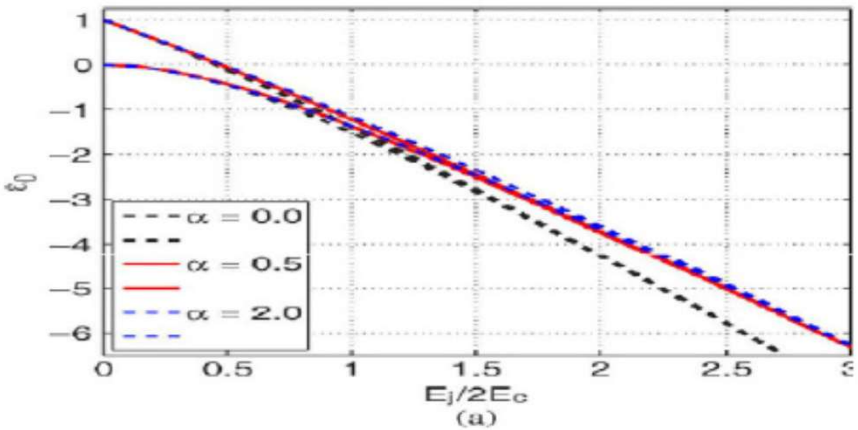


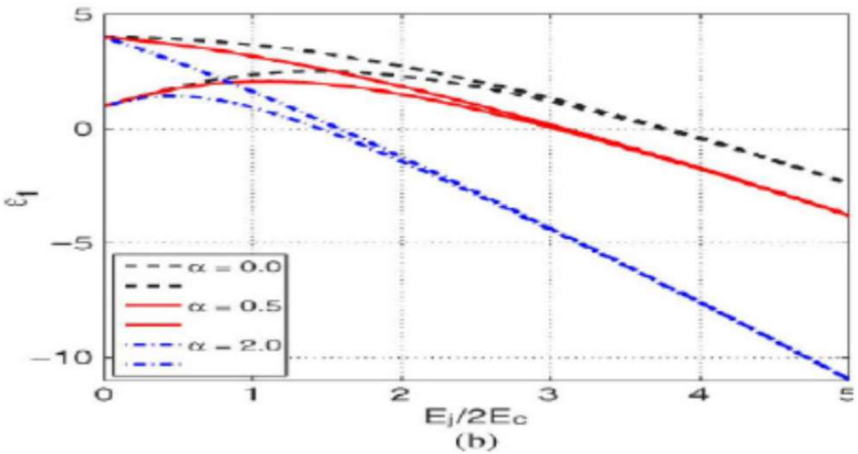
Canturk M., Askerzade I.N., IEEE Applied Superconductivity,23, 3541 (2011)

Spectrum of Josephson qubit with anharmonic CPR

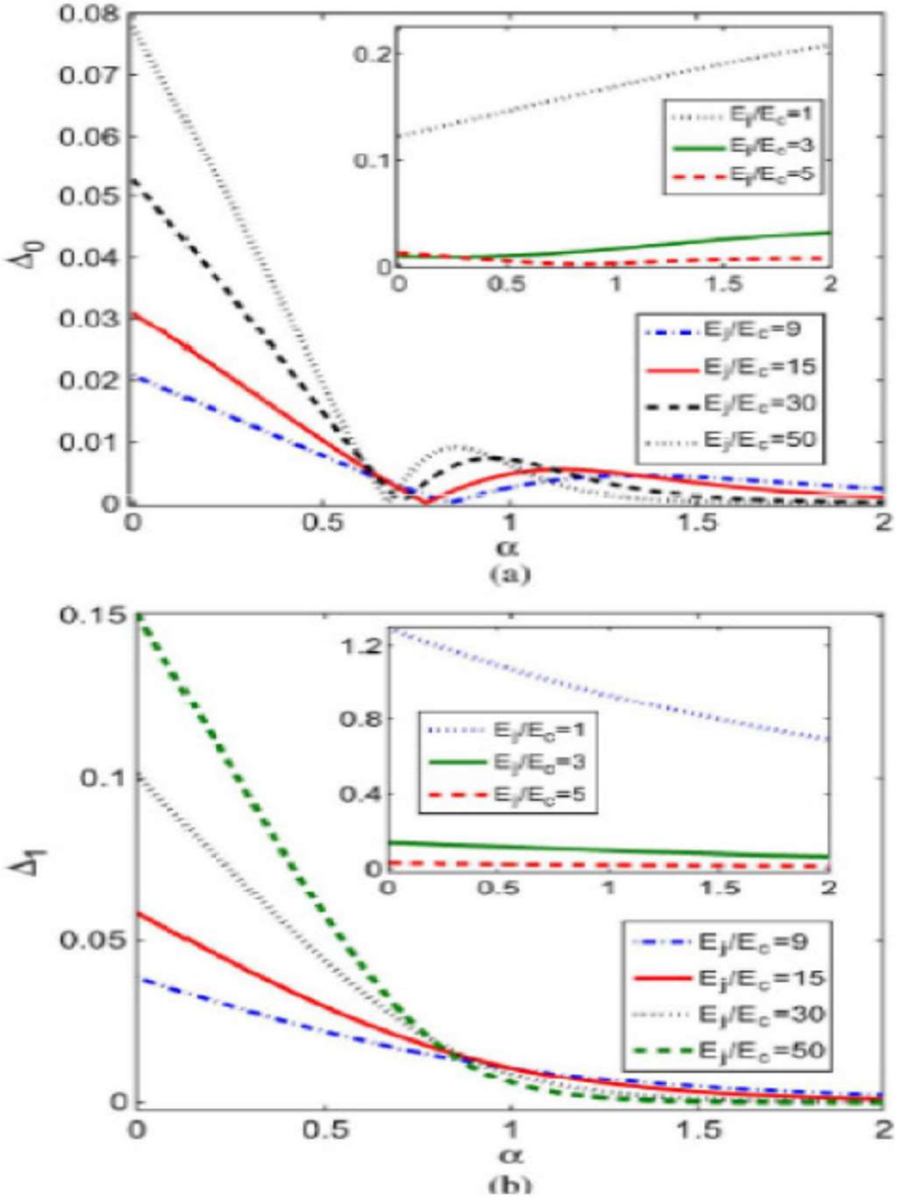
Spectrum of Mathieu equation





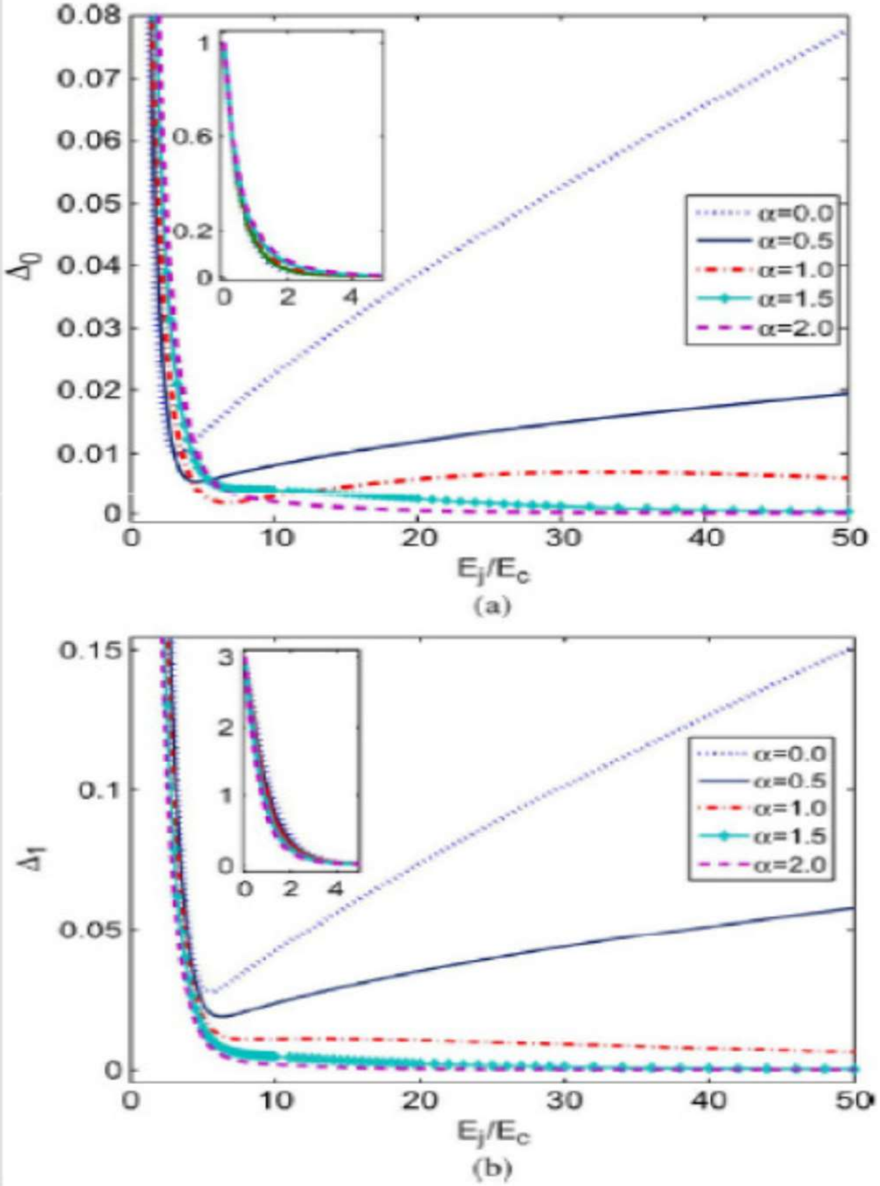


JJ phase qubit with anharmonic CPR



E_J/E_C	$\alpha_{\max}$	$\Delta_{0\,\max}$
9	1.350	0.0045
15	1.125	0.0054
30	0.950	0.0072
50	0.875	0.0090

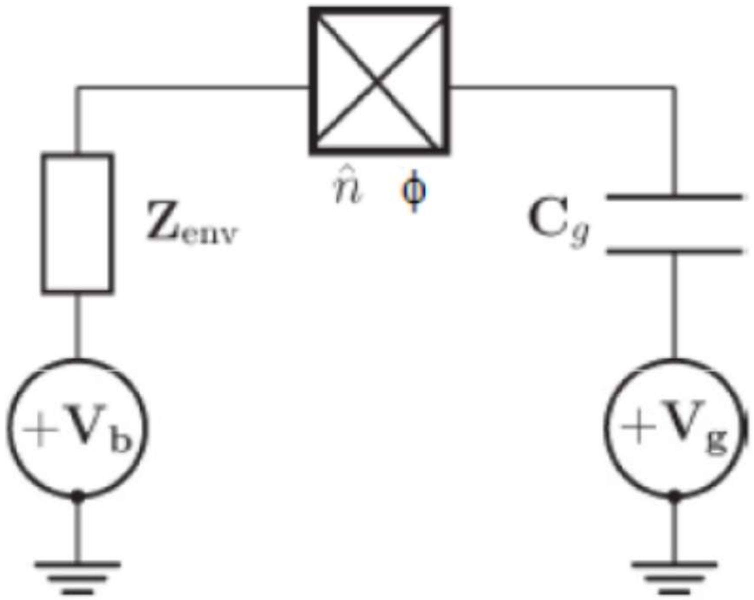
JJ phase qubit with anharmonic CPR



Charge qubits

Canturk M., Kurt E., Askerzade I.N., Int. J. Comput. Math. Elect. Electron. Eng, 30, 775(2011)

$$H = E_c(\hat{n} - n_g)^2 - E_j\left\{i_b\phi + \cos\phi - \frac{\alpha}{2}\cos 2\phi\right\}$$



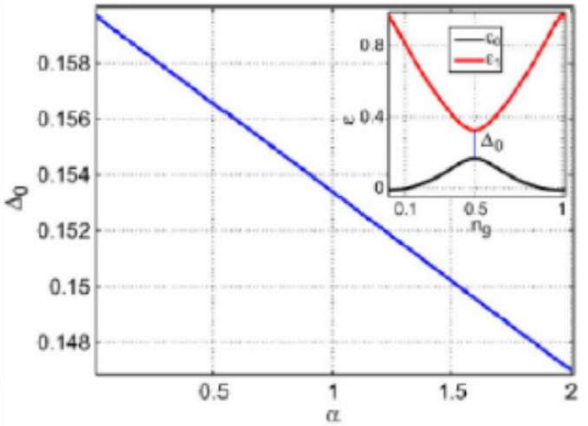
$$i_s = \langle\Psi|\, \hat{I}_s/I_c\, |\Psi\rangle = \frac{1}{b-a}\int_a^b \Psi^* \{\sin\phi - \alpha\sin 2\phi\}\, \Psi d\phi.$$

$$\langle\hat{n}\rangle = \langle\Psi|\, \hat{n}\, |\Psi\rangle = \frac{1}{b-a}\int_a^b \Im\left\{\Psi^* \frac{\partial\Psi}{\partial\phi}\right\} d\phi.$$

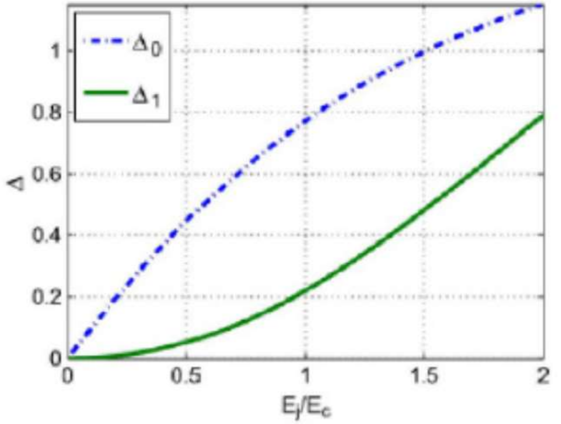
$$\mathbf{A}_0 = \begin{pmatrix} f_0 & e^* & 0 & 0 & 0 & \cdots & 0 & e \\ e & f_1 & e^* & 0 & 0 & \cdots & 0 & 0 \\ 0 & e & f_2 & e^* & 0 & \cdots & 0 & 0 \\ 0 & 0 & e & f_3 & e^* & \cdots & 0 & 0 \\ 0 & 0 & 0 & e & f_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & f_{N-2} & e^* \\ e^* & 0 & 0 & 0 & 0 & \cdots & e & f_{N-1} \end{pmatrix}$$

$$\mathbf{A}_0\Psi_n = \lambda_n\Psi_n, \quad n = 0, 1, 2, \ldots$$

Charge qubit with anharmonic CPR JJ

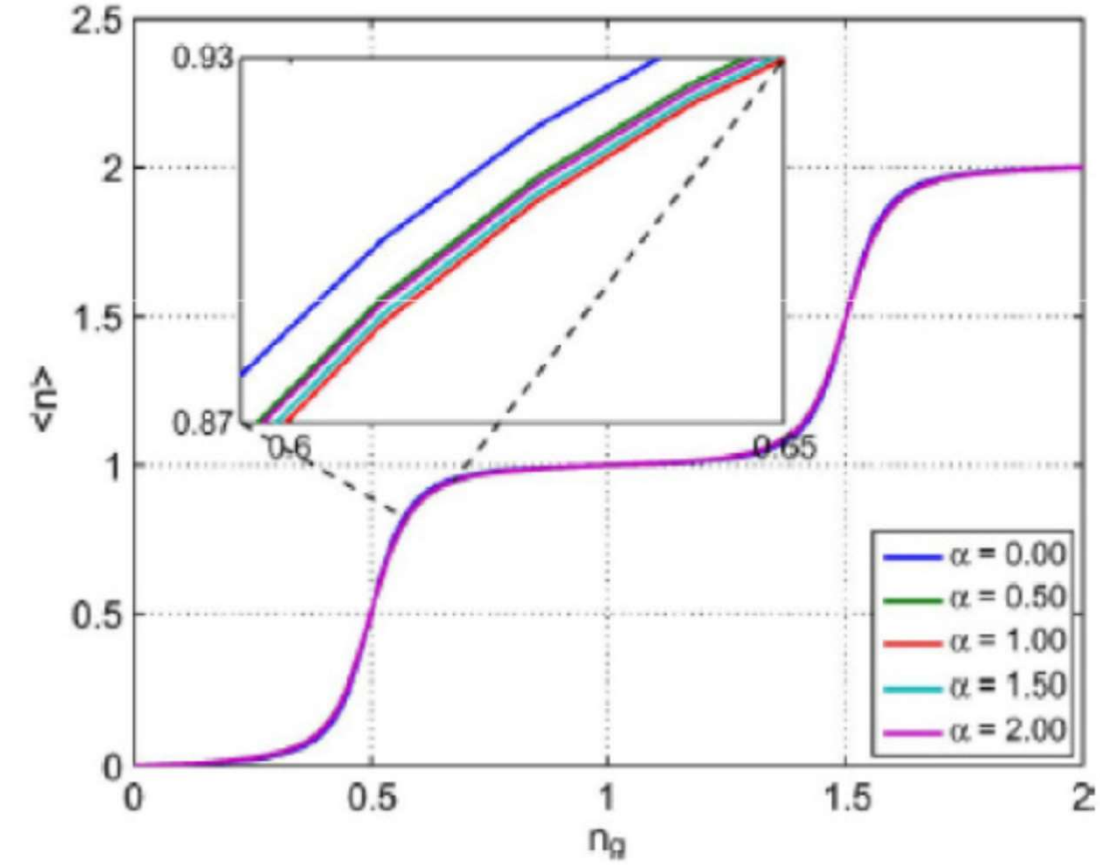


(a)

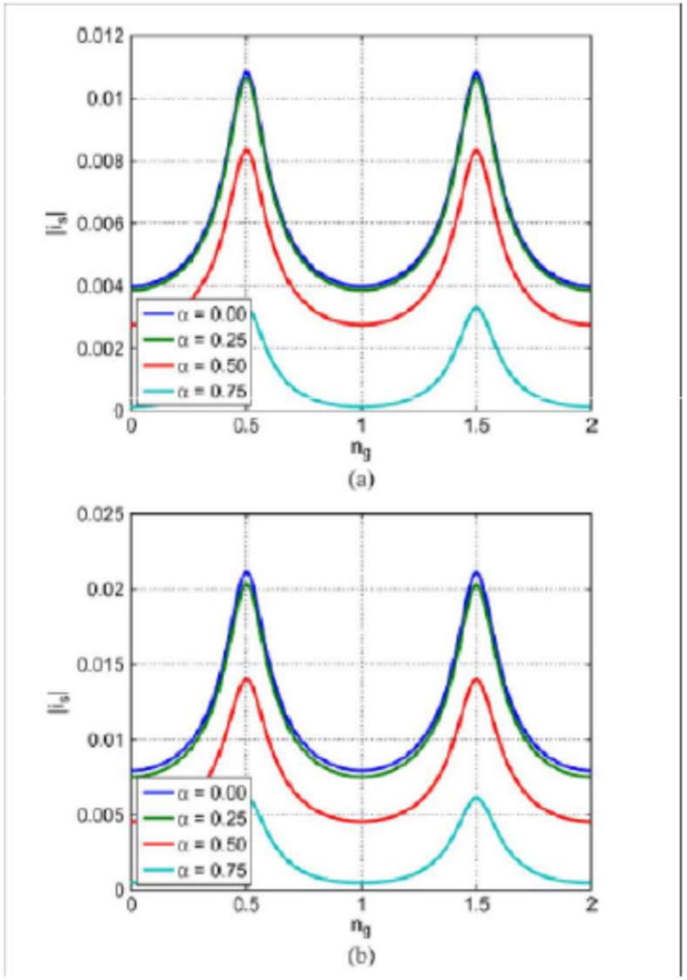


(b)

Charge qubits



Charge qubits



Conlusions

- Basic equations of Josephson Dynamics presented
- Order parameter symmetry and multiband character of new superconducting materials on current-phase-relation of JJ is analyzed
- Different types of superconducting qubits and their characteriscs is discussed
- Influence of anharmonic CPR on qubit spectrum is considered